Multiproduct Firms in Hotelling’s Spatial Competition∗

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Abstract

Oligopoly models are usually analyzed in the context of two firms, anticipating that market outcomes would be qualitatively similar in the case of three or more firms. The literature on Hotelling’s location-then-price competition is not an exception. In this paper, we show that the main finding of brand bunching in Hotelling’s duopoly no longer holds once three or more firms are allowed to enter the market. That is, in oligopoly with three or more firms, firms proliferate brands.

Keywords: price-then-location competition, multi-outlet oligopoly, brand proliferation, triopoly

JEL Classification: L13, R32

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1 Introduction

Literature on the oligopoly theory is often confined to two firms for the sake of analytical convenience and with the expectation that the number of firms would not qualitatively affect oligopolistic markets much. However, it is known in the literature on Hotelling’s (1929) spatial competition with quadratic transport costs that the equilibrium location is very different between duopoly and oligopoly with three or more firms. That is, if two firms compete first in terms of location and then price, they will be located as far as possible; this is interpreted as maximum differentiation in characteristic space. On the other hand, Neven (1987) and Brenner (2005) demonstrate that if there are more than two firms in the market, they do not maximize differentiation. Therefore, some of the established results on Hotelling’s duopoly no longer hold when more than two firms are allowed to enter the market.

This paper aims to settle the important debate in industrial organization—brand proliferation—by considering an oligopoly with more than two firms. We address the established findings of Martinez-Giralt and Neven (1988) that in the context of location-then-price competition duopolists do not open multiple outlets, or multiple brands in industrial organization, even if there are no fixed costs. The incentives to proliferate brands have long been attracting attention in the field of industrial organization. Schmalensee (1978) argued that incumbent firms deter new entry by brand proliferation in the ready-to-eat breakfast cereal industry, whereas Judd (1985) showed that such preemption is not credible in the absence of substantial exit costs. This is because an incumbent firm is better off by withdrawing some brands so that he avoids intense competition. It is true that Judd’s framework is general enough, but his conclusions are confined to duopoly. There is no guarantee that his conclusions hold for an oligopoly with more than two firms.

The purpose of this paper is to investigate whether brand proliferation is credible or not for oligopoly with more than two firms in the context of location-then-price competition. This paper analyzes multi-outlet triopoly, whereas Martinez-Giralt and Neven (1988) consider multi-outlet duopoly. This paper endogenizes locations of outlets whereas González-Maestre (2001) and Giraud-Héraud, Hammoudi and Mokrane (2003) focus on exogenous locations with equal distance.

Multi-outlet oligopoly can also be analyzed on the basis of different frameworks. First,
it can be examined under location-then-quantity competition a la Cournot. However, we
do not adopt this approach because equilibrium configurations do not necessarily fit the
reality. Price competition in this paper yields segmentation (firms serve a connected set of
types, minimizes competition with rival brands) and interlacing (firms serve disconnected
niches, maximize competition with rival brands), whereas the same does not hold for
quantity competition according to Pal and Sarkar (2002). Under quantity competition,
each firm opens outlets the way a monopolist would.

Second, rich equilibrium configurations might be obtained by assuming an elastic
demand for the good. However, works such as Economides (1984) indicate that obtaining
analytically meaningful results with elastic demand is too difficult. For example, despite
the identical assumptions of the model with elastic demand, Wang and Yang (1999) and
Rath and Zhao (2001) arrive at different equilibrium configurations when the reservation
price is low.

Third, diverse equilibrium configurations may also emerge by employing random utili-
ties such as logit models in the consumers’ choice of a firm (Chisholm and Norman, 2004).
However, obtaining analytically meaningful and interesting results seems impossible when
considering multiple outlets of more than two firms. We, therefore, focus on the tradi-
tional approach of Hotelling’s location-then-price competition due to its mathematically
tractability and because it provides a more accurate description of reality.

The general setting of Hotelling’s location-then-price competition model is explained
in the next section. We examine two kinds of consumer distributions. Section 3 considers
Hotelling’s original space, where consumers are uniformly distributed over a line segment,
and section 4 considers a simpler one, where consumers are uniformly distributed over a
circumference of a circle. We analyze both duopoly and triopoly with multiple outlets in
these two sections. We also investigate sequential entry under perfect foresight in section
4. We then conduct a brief empirical analysis in section 5. Section 6 concludes the paper.

2 General Setting

Consumers are uniformly distributed over a convex set in \( \mathbb{R} \), which is a line segment or a
circumference of a circle. Each consumer purchases one unit of an identical good. There
are $n$ firms with 2 outlets at the most, which are located on the convex set.\footnote{Firms may establish more than two outlets when the additional costs of outlets are low. This may be likely in the case of a characteristic space such as the colors of clothes. We restrict our analysis to the case of two outlets because the overall results do not qualitatively differ much even if the firms are allowed to produce many varieties (i.e., establish many outlets).} We ignore the cost of establishing outlets for analytical simplicity. They produce a good without cost and sell it at the mill price. Consumers bear the transport cost for shopping, which is quadratic in distance $x$ as given by $tx^2$.

The game in this paper is as follows. Each firm simultaneously determines the number and location of outlets in the first stage, and then they simultaneously select each mill price in the second stage. We know from Caplin and Nalebuff (1991) that there always exists a unique Nash price equilibrium in the second-stage price subgame under this setting, whereas there may be multiple equilibria in the first-stage location subgame. In this paper, we seek a sub-game perfect Nash equilibrium (SPNE).

Nash price equilibrium in the second stage is a price system where no firm wants to change prices of its two outlets given the number and location of outlets of the firms. More formally, it is defined by

$$\pi_i (p_i^*, p_{-i}^*) \geq \pi_i (p_i, p_{-i}^*) \quad \forall i = 1, \ldots, n \quad (1)$$

where $p_i \equiv (p_{i1}, p_{i2})$ is the price vector of firm $i$, $p_{-i}$ is the set of all price vectors except $i$, and $p_{i1}$ and $p_{i2}$ are the prices of firm $i$'s outlets 1 and 2.

The Nash location equilibrium in the first stage is a configuration where no firm wants either to relocate its outlets or to change their number. This is defined by

$$\pi_i (x_i^*, x_{-i}^*) \geq \pi_i (x_i, x_{-i}^*) \quad \forall x_i \text{ and } i = 1, \ldots, n \quad (2)$$

where $x_i \equiv (x_{i1}, x_{i2})$ is the location vector of firm $i$, $x_{-i}$ is the set of all location vectors except $i$, and $x_{i1}$ and $x_{i2}$ are the locations of firm $i$'s outlets 1 and 2. The number of outlets of firm $i$ is one if $x_{i1} = x_{i2}$, and two if $x_{i1} \neq x_{i2}$.

### 3 Line segment

In this section, consumers are uniformly distributed over the line segment with a unit length, $x \in [0, 1]$. 

\begin{equation}
\pi_i (p^*_i, p^*_{-i}) \geq \pi_i (p_i, p^*_{-i}) \quad \forall i = 1, \ldots, n
\end{equation}
3.1 Duopoly

Suppose each duopolist \( i = a, b \) is allowed to establish only one outlet. Then, d’Aspremont, Gabszewicz and Thisse (1979) show that both firms establish one outlet at the opposite ends of the line segment \((x_a^*, x_b^*) = (0, 1)\) in SPNE.\(^2\) What if firm \( b \) is allowed to open two outlets at \( x = x_{b1}, x_{b2} \) with \( x_a < x_{b1} < x_{b2} \leq 1 \) keeping \( x_a \) fixed? Because the full prices of the good in the visiting outlets \( a \) and \( b_1 \) are equal for marginal consumers locating at location \( \hat{x}_{ab} \),

\[
p_a + (\hat{x}_{ab} - x_a)^2 = p_{b1} + (x_{b1} - \hat{x}_{ab1})^2
\]

holds. Then, we get the market boundary between the two outlets as

\[
\hat{x}_{ab1} = \frac{p_{b1} - p_a}{2(x_{b1} - x_a)} + \frac{x_{b1} + x_a}{2}
\]

The other market boundary \( \hat{x}_{b1b2} \) is similarly obtained.

The profit of firm \( b \) is defined as

\[
\pi_b = p_{b1}(\hat{x}_{b1b2} - \hat{x}_{ab1}) + p_{b2}(1 - \hat{x}_{b1b2})
\]

Because this is quadratic and concave in \( p_{b1} \) and \( p_{b2} \), the first-order conditions are linear in \( p_{b1} \) and \( p_{b2} \), ensuring that the unique price equilibrium in the second-stage price subgame is unique and obtained explicitly. Plugging the equilibrium prices into the profits, they can be expressed as functions of locations \( x_{b1} \) and \( x_{b2} \). Solving the first-order conditions with respect to locations in the first stage, we have four candidates for SPNE. However, none of the candidates satisfies both the second-order conditions for profit maximization and \( x_{b1} < x_{b2} \). That is, firm \( b \) never establishes two outlets, and similarly for firm \( a \).

When both firms can open two outlets, there are three spatial arrangements: \( a_1b_1a_2b_2 \), \( a_1a_2b_1b_2 \) and \( a_1b_1b_2a_2 \) up to permutation. However, none of them satisfies some of the conditions for SPNE. Hence, we have the following.

\textbf{Proposition 1} When consumers are uniformly distributed over a line segment, there exists the unique SPNE such that duopolists open only one outlet and locate at opposite ends of a line segment.\(^3\)

\(^2\)If firms are allowed to establish outside \([0, 1]\), then the SPNE is given by \((x_a^*, x_b^*) = (-1/4, 5/4)\) (Tabuchi and Thisse, 1995).

\(^3\)Proofs of the statements in this paper, which are often long and tedious, may be obtained from the author upon request.
Thus, firms locate as far as possible from each other and do not open multiple outlets in order to mitigate the intense price competition. This implies that relaxing product differentiation is maximized in the characteristic space in the case of duopoly. In other words, under duopolistic location-then-price competition, price competition takes precedence over increasing the market reach. However, we can show in the next section that such maximum differentiation is not true for triopoly. In contrast to the case of a duopoly, firms have an incentive to open multiple outlets in the case of a triopoly.

3.2 Triopoly

Suppose next there are three firms \( i = a, b, c \), each of which establishes one outlet. Then, Brenner (2005) shows that there exists a unique SPNE locations given by \( (x^*_a, x^*_b, x^*_c) = (1/8, 4/8, 7/8) \). Observe that neither maximum differentiation nor minimum differentiation holds in the triopolistic market.

First, what if firms are allowed to open multiple outlets? Consider the case that firm \( c \) deviates from one to two outlets at \( x = x_{c1}, x_{c2} \) with \( 4/8 < x_{c1} < x_{c2} \leq 1 \) while firms \( a \) and \( b \) remain unchanged, i.e., \( (x^*_a, x^*_b) = (1/8, 4/8) \) are fixed. Computing the first-order conditions in the second-stage price competition, we have the first-order conditions in the first-stage location competition for firm \( c \) as

\[
x_{c2} = \frac{2 + x_{c1}}{3}
\]

\[
163840x_{c1}^3 + 53248x_{c1}^4 - 97280x_{c1}^3 + 28288x_{c1}^2 - 67640x_{c1} - 5639 = 0
\]

Solving them yields the unique solution:

\( (x^*_c) \simeq (0.856, 0.952) \)

with a higher profit. This implies that firm \( c \) has an incentive to open two outlets, and so does firm \( a \) due to axisymmetry. Insofar as firms have incentives to open multiple outlets, the single outlet triopoly \( (x^*_a, x^*_b, x^*_c) = (1/8, 4/8, 7/8) \) is not an SPNE.\(^4\) Second, consider deviation of firm \( b \) from one to two outlets \( x = x_{b1}, x_{b2} \) with \( 0 \leq x_{b1} < x_{b2} \leq 1 \) while keeping \( (x^*_a, x^*_c) = (1/8, 7/8) \) fixed? Then, we can show that firm \( b \) never establishes two outlets unlike firms \( a \) and \( c \) because some of the conditions for SPNE are violated.

\(^4\)The property that triopolists have incentives to establish multiple outlets is also confirmed when consumers are uniformly distributed over a two-dimensional disk (Tabuchi, 2009).
These results may suggest that there exists an SPNE in which the central firm opens one outlet and the peripheral firms open two outlets. In fact, it can be shown that if each firm is allowed to open up to two outlets, then

\[
(x_1^*, x_2^*, x_b^*, x_c1^*, x_c2^*) = \left( \frac{11 - \sqrt{73}}{48}, \frac{11 - \sqrt{73}}{16}, \frac{5 + \sqrt{73}}{16}, \frac{37 + \sqrt{73}}{48} \right)
\]

is an SPNE. The corresponding prices are

\[
(p_1^*, p_2^*, p_b^*, p_c1^*, p_c2^*) \simeq (0.201, 0.191, 0.156, 0.191, 0.201)
\]

which exhibits a U-shaped price structure.\(^5\) The profits are calculated as

\[
(\pi_a^*, \pi_b^*, \pi_c^*) \simeq (0.053, 0.070, 0.053)
\]

indicating the locational advantage at the center. That is, the central firm \(b\) has the highest profit although it opens only one outlet with the lowest price, and although it seems to be the most embattled by competition.

The above SPNE is a segmented configuration. We can similarly show that the interlacing configuration

\[
(x_1^*, x_b1^*, x_c1^*, x_a2^*, x_b2^*, x_c2^*) \simeq (0.062, 0.253, 0.410, 0.590, 0.747, 0.938)
\]

is also an SPNE. The corresponding prices exhibit U-shaped and the profit of \(b\) is higher than that of \(a\) and \(c\). Thus, there exist multiple equilibria in the first-stage location subgame although there is a unique equilibrium in the second-stage price subgame.

The intuition would be as follows. In the case of duopoly, opening multiple outlets (brand proliferation) is a big negative due to intensifying price competition. However, in the case of triopoly, price competition is dominated by the positive aspect of multiple outlets (being able to capture another niche with interlacing or charging multiple prices at multiple locations with segmenting). When there is a third firm, the advantage of reducing the average distance from consumers by “moving inward” outweighs the disadvantage of reducing the distance from competitors.

\(^5\)Such a U-shape is also obtained in the case of five single-outlet firms by Economides (1993) and Brenner (2005).
There are 90 spatial arrangements if triopolists open two outlets. Removing the same permuted arrangements such as $a_1 a_2 b_1 b_2 c_1 c_2$ and $c_1 c_2 b_1 b_2 a_1 a_2$, there are still 12 spatial arrangements. Therefore, we stop searching for other SPNE. In what follows, we change the consumer distribution from the unit line segment to the unit circumference of a circle in order to exploit analytical tractability due to locational symmetry.

4 Circumference of a circle

In this section, consumers are uniformly distributed over the circumference of a circle with a unit length. The location space is denoted by $x \in [0, 1]$ along the circumference of a circle, where $x = 0$ coincides with $x = 1$.

4.1 Duopoly

There are two firms, each of which can open two outlets. Martinez-Giralt and Neven (1988) showed that firms do not open multiple outlets in a duopoly and, further, that both firms establish one outlet at opposite ends of a diameter of a circle, assuming quasi-symmetric location patterns, where “outlets (whoever owns them) are positioned at opposite ends of two diameters”. Although they did not rule out the existence of asymmetric equilibria, we can show nonexistence of such asymmetric equilibria and uniqueness of the following SPNE up to rotation.

Proposition 2 When consumers are uniformly distributed over a circumference of a circle, there exists the unique SPNE up to rotation such that duopolists open only one outlet and locate at opposite ends of a diameter.

The proof is contained in Appendix 1. Observe the similarity between Propositions 1 and 2 although the consumer distributions are different. In both distributions, duopolists relax product differentiation by locating far away.

The nonexistence of asymmetric equilibria is intuitive because of the location symmetry of the model setting. Nevertheless, we will see below that there exists an asymmetric SPNE in the case of triopoly.
4.2 Triopoly

Consider triopolists $i = a, b, c$ who are allowed to establish only one outlet. Then, there exists an SPNE given by $(x^*_a, x^*_b, x^*_c) = (0, 1/3, 2/3)$ from Theorem 5 in Economides (1989). Furthermore, its uniqueness up to rotation is readily verified. This is because the best locational reply of each firm in the first stage is shown to be a midpoint of the neighboring firms. Therefore, in the case of a single outlet, the principle of maximum differentiation holds both in a duopoly and triopoly.

Suppose firm $c$ is allowed to open two outlets at $x = x_{c1}, x_{c2}$ with $1/3 \leq x_{c1} < x_{c2} \leq 1$ given $(x^*_a, x^*_b) = (0, 1/3)$. Computing the first-order conditions and checking the second-order conditions in the second and first stages, we can show that firm $c$ has an incentive to open two outlets. The SPNE locations are determined by the following equations

$$x_{c1} + x_{c2} = \frac{4}{3}$$
$$54x_{c1}^3 - 15x_{c1}^2 - 7x_{c1} - 3 = 0$$

whose real solution is unique and given by

$$(x^*_{c1}, x^*_{c2}) \approx (0.626, 0.707)$$

The profit of firm $c$ with two outlets is shown to be strictly higher than that with one outlet. Furthermore, due to the location symmetry of a circle, both firms $a$ and $b$ would also have incentives to establish two outlets. Insofar as a firm has an incentive to open multiple outlets, the single-outlet triopoly $(x^*_a, x^*_b, x^*_c) = (0, 1/3, 2/3)$ is not an SPNE. This conclusion is the same as that with the line segment in section 3.2.

Assume next that each triopolist opens two outlets. Unlike the case of the line segment, we can reduce the number of spatial arrangements further in the case of the circumference of a circle by exploiting location symmetry. Removing the same permuted arrangements, we can verify that there are only 5 spatial arrangements up to permutation and rotation as follows:

- **interlacing** $a_1b_1c_1a_2b_2c_2$
- **partial segmentation** $a_1b_1b_2a_2c_1c_2$
- **segmentation** $a_1a_2b_1b_2c_1c_2$
- **quasi-interlacing** $a_1b_1c_1a_2c_2b_2$
- **quasi-partial segmentation** $a_1b_1c_1c_2a_2b_2$
Unlike the duopoly case, we cannot show a unique SPNE given each spatial arrangement. However, using the damped Newton’s method in Mathematica calculations with several initial values of locations, we find that each spatial arrangement yields a unique SPNE candidate. As shown in Appendix 2, the first three spatial arrangements are SPNE, whereas the last two are not. Quasi-interlacing is shown not to be an SPNE because withdrawing one outlet of firm $a$ is shown to raise its profit. Quasi-partial segmentation is shown not to be an SPNE because relocating an outlet of firm $c$ to location between, say, $a_1$ and $b_2$ is shown to increase its profit. In sum, we establish the following.

**Proposition 3** There exists the SPNE as follows:

(i) interlacing: $(x_{a_1}^*, x_{b_1}^*, x_{c_1}^*, x_{a_2}^*, x_{b_2}^*, x_{c_2}^*) = \left(0, \frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}\right)$,

(ii) partial segmentation: $(x_{a_1}^*, x_{b_1}^*, x_{b_2}^*, x_{a_2}^*, x_{c_1}^*, x_{c_2}^*) = \left(0, \frac{3}{16}, \frac{5}{16}, \frac{8}{16}, \frac{11}{16}, \frac{13}{16}\right)$,

(iii) segmentation: $(x_{a_1}^*, x_{a_2}^*, x_{b_1}^*, x_{b_2}^*, x_{c_1}^*, x_{c_2}^*) = \left(\frac{1}{6} - r, \frac{1}{6} + r, \frac{3}{6} - r, \frac{3}{6} + r, \frac{5}{6} - r, \frac{5}{6} + r\right)$,

where $r = \left(11 - \sqrt{73}\right) / 36 \approx 0.068$.

The spatial configurations are illustrated in Figure 1. This proposition shows multiple equilibria in the first-stage location subgame as in the case of the line segment. Because each consumer demands one unit of a good inelastically, the sum of the consumers’ expenditure coincides with the sum of the firms’ revenues. This implies that minimizing the total transport costs is equivalent to maximizing the social welfare. Therefore, the interlacing is the socially optimal configuration.

The corresponding profits of the three configurations are computed as follows:

(i) interlacing: $\pi_i^* = t/108$ for $i = a, b, c$,

(ii) partial segmentation: $\pi_a^* = 242t/12288 > 169t/12288 = \pi_b^* = \pi_c^*$,

(iii) segmentation: $\pi_i^* = (1 - 3r)t/27$ for $i = a, b, c$.

The interlacing profit is the lowest and the segmentation profit is the highest, meaning that the proximity to competitors is crucial for firms. If other firms are located apart, local monopoly is possible by market segmentation.

Presume that a fourth firm enters and opens one outlet when the multi-outlet incumbents are located as in Proposition 3. Then, this profit maximizing firm would locate its outlet at a location that is the most distant from incumbent outlets. The outlet location candidates of the fourth firm are $x = 1/12$, $3/32$, and $1/3$ in cases (i), (ii), and (iii), respectively. It can be shown that the profit of the entrant is the smallest ($0.0014t$) in
case (i) of interlacing configuration and the largest (0.0069t) in case (iii) of segmentation. This is parallel to the profits of the incumbents, which are the smallest (0.0093t) in the interlacing case and largest (0.0295t) in the segmentation case. We may therefore state that keen competition reduces the profits of incumbents but is likely to prevent the entry of new firms. If there exists a fixed cost for entry, which is slightly larger than 0.0014t, then both the segmentation and the partial segmentation are vulnerable to the entry of a fourth firm, whereas the interlacing is not. There are rich dynamic implications: the interlacing configuration is most effective at preventing entry, but this comes at the cost of relatively low profits for incumbents.

Finally, the results of Proposition 3 may be extended to a larger number of firms. For example, if there are four firms on the circumference of a circle, then applying the proof in Appendix 2 we can verify that both interlacing

\[
(x_{a1}^*, x_{b1}^*, x_{c1}^*, x_{d1}^*, x_{a2}^*, x_{b2}^*, x_{c2}^*, x_{d2}^*) = \left(0, \frac{1}{8^*}, \frac{2}{8^*}, \frac{3}{8^*}, \frac{4}{8^*}, \frac{5}{8^*}, \frac{6}{8^*}, \frac{7}{8^*}\right)
\]

and segmentation

\[
(x_{a1}^*, x_{a2}^*, x_{b1}^*, x_{b2}^*, x_{c1}^*, x_{c2}^*, x_{d1}^*, x_{d2}^*) = \left(\frac{1}{8} - q, \frac{1}{8} + q, \frac{3}{8} - q, \frac{3}{8} + q, \frac{5}{8} - q, \frac{5}{8} + q, \frac{7}{8} - q, \frac{7}{8} + q\right)
\]

are SPNE, where \(q = (31 - 3\sqrt{57})/224 \approx 0.037\).

### 4.3 Sequential entry

Thus far, we have been considering simultaneous entry of firms. What if firms enter sequentially under perfect foresight a la Prescott and Visscher (1977) keeping the assumption of the uniform distribution of consumers over the unit circumference of a circle? In the case of duopoly, firm A enters and opens at most two outlets in the first stage, then firm B enters and opens at most two outlets, and both firms compete in prices. It can be readily verified that the sequential locations coincide with the simultaneous ones. That is, Proposition 2 holds: both firms establish only one outlet and locate at opposite ends of a diameter.

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6This raises hopes that the level of the fixed cost may be used as an equilibrium selection criterion. For example, the interlacing configuration is more likely to emerge under a small fixed cost for entry, while segmentation tends to appear under a larger fixed cost. However, we are unable to get clear results due to the presence of multiple equilibria even with the fixed cost.
In the case of triopoly, firm $A$ enters and opens at most two outlets, then firm $B$ does the same, then firm $C$ does the same, and finally they compete in prices. The price subgame in the fourth stage is uniquely determined as before. In the sequel, we focus on the first three stages.

Let $\bar{\pi}_i$ be the profit of firm $i$ after substituting the equilibrium prices. The best replies of firm $C$ in the third stage are determined by the first-order conditions

$$f_{ck} \equiv \frac{\partial \bar{\pi}_c (x_{c1}, x_{c2})}{\partial x_{ck}} = 0$$

for $k = 1, 2$. Next, the best replies of firm $B$ in the second stage are given by the first-order conditions

$$f_{bk} \equiv \frac{d \bar{\pi}_b (x_{b1}, x_{b2}, x_{c1} (x_{b1}, x_{b2}), x_{c2} (x_{b1}, x_{b2}))}{dx_{bk}} = \frac{\partial \bar{\pi}_b}{\partial x_{bk}} + \sum_{l=1}^{2} \frac{\partial \bar{\pi}_b}{\partial x_{cl}} \frac{\partial x_{cl}}{\partial x_{bk}} = 0$$

for $k = 1, 2$, where $\partial x_{cl}/\partial x_{bk}$ are computed by applying the implicit function theorem to $f_{c1} = f_{c2} = 0$. We denote $x_{ck} (x_{b1}, x_{b2})$ in the above because in locating outlets firm $B$ takes the subsequent locations of firm $C$’s outlets into account. Finally, set $x_{a1} = 0$ without loss of generality. Then, the best reply of firm $A$ in the first stage is

$$f_{a2} \equiv \frac{d \bar{\pi}_a (x_{a2}, x_{b1} (x_{a2}), x_{b2} (x_{a2}), x_{c1} (x_{b1} (x_{a2}), x_{b2} (x_{a2})), x_{c2} (x_{b1} (x_{a2}), x_{b2} (x_{a2}))))}{dx_{a2}}$$

$$= \frac{\partial \bar{\pi}_a}{\partial x_{ak}} + \sum_{l=1}^{2} \frac{\partial \bar{\pi}_a}{\partial x_{bl}} \frac{\partial x_{bl}}{\partial x_{ak}} + \sum_{l=1}^{2} \sum_{m=1}^{2} \frac{\partial \bar{\pi}_a}{\partial x_{cl}} \frac{\partial x_{cl}}{\partial x_{bm}} \frac{\partial x_{bm}}{\partial x_{ak}} = 0$$

where $\partial x_{bl}/\partial x_{ak}$ and $\partial x_{bm}/\partial x_{ak}$ are computed by applying the implicit function theorem to $f_{b1} = f_{b2} = 0$ and $\partial x_{cm}/\partial x_{bk}$ are computed by applying the implicit function theorem to $f_{c1} = f_{c2} = 0$.

Thus, we have the five first-order conditions $f_{a2} = f_{b1} = f_{b2} = f_{c1} = f_{c2} = 0$ and the five unknowns $x_{a2}, x_{b1}, x_{b2}, x_{c1}$ and $x_{c2}$. Solving them simultaneously by using the damped Newton’s method in Mathematica for all spatial arrangements with various initial values, we numerically confirm that market segmentation is an SPNE, whose locations are given by

$$(x_{a1}^s, x_{a2}^s, x_{b1}^s, x_{b2}^s, x_{c1}^s, x_{c2}^s) \approx (0, 0.262, 0.508, 0.706, 0.852)$$

Note that the SPNE in the sequential game is unique up to rotation unlike the simultaneous game in the foregoing sections. Whereas the market segmentation is the unique outcome under sequential entry, we have seen in the previous section that the interlacing
configuration is robust against further entry of firms. This implies that the SPNE spatial arrangements are subject to the timing of entry.

Earlier entrants try to segment the market with perfect foresight in order to seek higher profits. In fact, the first entrant establishes two outlets whose distance is the largest (0.262), the second entrant establishes two outlets, and the last entrant establishes only one outlet ($x_{c1} = x_{c2}$). Incumbents can acquire larger demand in advance by keeping distance between two outlets. As a result, there is no room left for the last entrant to open multiple outlets. This may give an explanation on the fact that incumbents offer more product varieties than new entrants in the real world (Schmalensee, 1978).

The equilibrium profits are calculated as

$$(\pi^*_a, \pi^*_b, \pi^*_c) \simeq (0.028t, 0.023t, 0.011t)$$

The first entrant gets the highest profit, the second entrant earns less profit, and the last entrant the least. This is so-called the first-mover advantage. Note that these profits under sequential entry are higher or lower than those under simultaneous entry in the previous section: $t/108 \simeq 0.009t$ in the interlacing configuration and $(1 - 3r)t/27 \simeq 0.029t$ in the segmented configuration. Therefore, even though the first entrant can secure a higher profit than the subsequent entrants, the first entrant is not necessarily able to receive a higher profit than that under simultaneous entry.\(^7\)

Finally, the equilibrium prices are computed as

$$(p^*_a, p^*_b, p^*_c) \simeq (0.063t, 0.083t, 0.075t, 0.058t, 0.041t)$$

They are related to distances to neighboring firms: $p^*_a$ is the highest because outlet $a_2$ is the most distant; and $p^*_c$ is the lowest because the distances from outlet $c$ to the neighboring outlets are the shortest. This is consistent with the findings in the literature on price-then-location competition. Casual empiricism suggests that new entrants tend to charge lower prices than incumbents, which is also in accord with the result of this sequential game.

\(^7\)This is in contrast to single-outlet triopoly on a line segment. The profits under simultaneous entry are $(\pi^*_a, \pi^*_b, \pi^*_c) \simeq (0.078t, 0.055t, 0.055t)$ where $(x^*_a, x^*_b, x^*_c) = (1/2, 1/8, 7/8)$ according to Brenner (2005), whereas those under sequential entry are $(\pi^*_a, \pi^*_b, \pi^*_c) \simeq (0.088t, 0.069t, 0.047t)$ where $(x^*_a, x^*_b, x^*_c) = (0.426, 0.889, 0.074)$ from Götz (2005).
5 Average distance

In order to test the foregoing theoretical findings, we have surveyed and collected the location data of convenience stores around major Japanese stations. Figure 2 illustrates the locations of convenience stores in March 2009 inside the circle with radius one kilometer around Shinjuku station in Tokyo. There are five firms—Seven-Eleven, Lawson, FamilyMart, am/pm, and CircleKSunkus—with 15, 18, 17, 25, and 17 stores, respectively.\(^8\) Similar location configurations are observed inside the circle with radius one kilometer around Shibuya station in Tokyo, Sakae station in Nagoya, and Nanba station in Osaka. The total number of convenience stores inside the circle in Shinjuku, Shibuya, Sakae, and Nanba are 92, 65, 65, and 67, respectively.

There is no doubt that convenience stores are typically multi-outlet spatial oligopolists. Although convenience stores sell a wide variety of products, their selections of varieties are quite similar. Therefore, goods are more or less homogeneous, which suggests that Hotelling’s spatial competition may apply. Convenience stores compete in the number and location of outlets. They normally sell goods at regular prices. However, they often compete in price in terms of, for example, discounts on plastic bottles of water and sandwiches.

In order to test whether the locations of convenience stores are interlaced or segmented, we compute the ratio of the average distance between outlets belonging to the same firm, \(d_s\), to that belonging to different firms, \(d_d\). We refer to \(d_s/d_d\) as the degree of mixing, which is independent in distance units. We can readily calculate the degree of mixing in each configuration appeared in Proposition 3 in section 4.2.\(^9\) However, because the number of firms are more than 2 and the number of outlets are large in the above sample data, we compute the degree of mixing with 5 firms having an infinite number of outlets in Appendix 3. The theoretical values of the degree of mixing \(d_s/d_d\) is shown to be 1 in the case of interlacing configuration and is 0.065 in the case of segmentation. The degree of mixing in the case of partial segmentation would be between these extreme values.

\(^8\)The numbers of stores in Figure 2 are smaller than these numbers because close stores are not displayed separately.

\(^9\)The degree of mixing \(d_s/d_d\) is 2 in the case of interlacing configuration, 0.8 in the case of partial segmentation, and 0.2 in the case of segmentation.
The empirical values can be calculated by measuring all the distances between convenience stores around Shinjuku, Shibuya, Sakae, and Nanba. They are shown to be $d_s/d_d \simeq 1.02, 0.98, 1.04$, and $1.03$, respectively, which are close to $1$ and exhibit strong similarity among them. Comparing the empirical values with the theoretical values, we may say that the spatial arrangements of convenience stores around the four big stations in Japan are the interlacing configuration due to entry deterrence by incumbents stated in the previous section. Note that this conclusion may be with reservation because theoretical values are for three firms around a circle with two outlets, whereas the empirical ones are for five firms inside the circle with 12-25 outlets.

6 Conclusion

We revisited Hotelling’s location-then-price competition by considering uniform distributions of consumers over a line segment and a circumference of a circle in order to settle the debates on brand proliferation. Comparing the market outcomes between duopoly and oligopoly with three or more firms under both simultaneous entry and sequential entry, we showed that firms proliferate brands in oligopoly with three or more firms but not in duopoly. We may therefore conclude that duopoly substantially differs from oligopoly with three or more firms.

We also conducted a brief empirical analysis in order to find the difference between theory and reality. Computing the degree of mixing, we have shown that the spatial configurations of convenience stores near big stations in Japan are close to the interlacing configurations. This may suggest that the location decision of convenience stores may be determined by entry deterrence because the interlacing configuration is most effective at preventing entry.

However, much work remains to be done in order to fully settle the above two debates. Our analysis has dismissed the elastic demand for the good. As we mentioned in the introduction, rich equilibrium configurations may be obtained by assuming an elastic demand although it is difficult to obtain SPNE under intermediate reservation prices. We have also dismissed the fixed costs of entry, which affect the entry of firms, the number of outlets, and the entry deterring location strategy (Bonanno, 1987). These market outcomes are perhaps more complicated but, nevertheless, richer and may better fit the
Appendix 1: Proof of Proposition 2

Firm \(i = (a, b)\) establishes outlets \(i_1\) and \(i_2\) at \(x = x_{i1}, x_{i2}\). The number of outlets of firm \(i\) is 1 if \(x_{i1} = x_{i2}\) and 2 if \(x_{i1} \neq x_{i2}\). Suppose outlets of firm \(i\) are located such that \(x_j < x_{i1} < x_k\) and \(x_l < x_{i2} < x_m\), where \(j, k, l, m\) could be \(i_1\) or \(i_2\). Then, the full prices of the good in the visiting outlets \(i_1\) and \(j\) are equal at location \(x_{i1j}\) of marginal consumers:

\[
p_{i1} + (\hat{x}_{i1j} - x_{i1})^2 = p_j + (x_j - \hat{x}_{i1j})^2
\]

which leads to the market boundary between outlets \(i\) and \(j\)

\[
\hat{x}_{i1j} = \frac{p_j - p_{i1}}{2(x_j - x_{i1})} + \frac{x_j + x_{i1}}{2}
\]

The other market boundaries are similarly computed. Then, the profit of firm \(i\) is defined as

\[
\pi_i = p_{i1}(\hat{x}_{i1k} - \hat{x}_{i1j}) + p_{i2}(\hat{x}_{i2l} - \hat{x}_{i1m})
\]

Because \(\pi_i\) is quadratic and concave in \(p_{i1}\) and \(p_{i2}\), the first-order condition is linear in \(p_{i1}\) and \(p_{i2}\), ensuring that the unique equilibrium prices in the second stage are explicitly obtained. Plugging the equilibrium prices into the profits, they can be expressed as functions of locations \(\pi_i(x_{a1}, x_{a2}, x_{b1}, x_{b2})\). Solving the first-order conditions with respect to locations yields the necessary conditions for SPNE.

There are two spatial configurations up to rotation and permutation:

\[
\begin{cases}
\text{interlacing} & 0 = x_{a1} \leq x_{b1} \leq x_{a2} \leq x_{b2} \leq 1 \\
\text{segmentation} & 0 = x_{a1} \leq x_{a2} \leq x_{b1} \leq x_{b2} \leq 1
\end{cases}
\]

where we fix \(x_{a1} = 0\) due to symmetry of the model.

**Interlacing**

Differentiating the profit \(\pi_i(0, x_{a2}, x_{b1}, x_{b2})\) with respect to \(x_{i1}\) and \(x_{i2}\) for \(i = a, b\) in the first stage, we get three first-order conditions for SPNE:

\[
\frac{\partial \pi_a}{\partial x_{a2}} = \frac{\partial \pi_b}{\partial x_{b1}} = \frac{\partial \pi_b}{\partial x_{b2}} = 0
\]
They can be solved by using the Buchberger’s algorithm (Cox, Little and O’Shea, 1997). One of the Gröbner bases is shown to be a 68th-order single-variable polynomial of $x_{b2}$ with integer coefficients. Computing all the solutions by the Jenkins-Traub algorithm, there are 9 solutions that are in the interval of $(0, 1)$. For each solution, we solve $\partial \tilde{\pi}_a / \partial x_{a2} = \partial \tilde{\pi}_b / \partial x_{b1} = 0$ for $x_{a2}$ and $x_{b1}$. Then, we can show that $x_{a2} = 3/4$ is the only solution that satisfies $0 \leq x_{b1} \leq x_{a2} \leq 3/4$. Hence, the unique SPNE candidate is given by $(x_{a1}, x_{a2}, x_{b1}, x_{b2}) = (0, 2/4, 1/4, 3/4)$.

However, because $\tilde{\pi}_a (0, 2/4, 1/4, 3/4) = t/32 < 25t/576 = \tilde{\pi}_a (0, 0, 1/4, 3/4)$, firm $a$ has an incentive to withdraw one of the two outlets. That is, there is no interlacing SPNE.

Segmentation

Similar to the interlacing case, we solve the three simultaneous equations using the Buchberger’s algorithm. One of the Gröbner bases is shown to be a 43th-order single-variable polynomial of $x_{b2}$. Computing all the solutions, there are 4 solutions that are in the interval of $(0, 1)$. However, none of them satisfies $0 \leq x_{a2} \leq x_{b1} \leq x_{b2}$ when we solve $\partial \tilde{\pi}_a / \partial x_{a2} = \partial \tilde{\pi}_b / \partial x_{b1} = 0$ for $x_{a2}$ and $x_{b1}$. Hence, there is no segmented SPNE.

Single versus two outlets

One of the firms withdraws one of the outlets. Consider the case that firm $a$ opens one outlet and firm $b$ opens two outlets, where $x_a \leq x_{b1} \leq x_{a2}$. As shown by the proof of Proposition 1 in Martinez-Giralt and Neven (1988), firm $b$ opens only one outlet and locates it at an opposite end of a diameter $x_b = x_a \pm 1/2$.

Appendix 2: Proof of Proposition 3

Firm $i = a, b, \text{and} c$ establishes outlets $i_1$ and $i_2$ at $x = x_{i1}, x_{i2}$. The market boundaries $\hat{x}_{ij}$ and the firm’s profits $\pi_i$ are determined by the beginning of Appendix 1. Solving the first-order conditions in price competition, plugging the equilibrium prices into the profits, and solving the first-order conditions in location competition, we obtain the necessary
conditions for SPNE. There are five SPNE candidates up to rotation and permutation:

\[
\left\{
\begin{array}{l}
\text{interlacing} & (x_{a1}, x_{b1}, x_{c1}, x_{a2}, x_{b2}, x_{c2}) = (0, \frac{1}{3}, \frac{2}{5}, \frac{2}{5}, \frac{4}{5}, \frac{5}{5}) \\
\text{partial segmentation} & (x_{a1}, x_{b1}, x_{b2}, x_{c1}, x_{c2}) = (0, \frac{3}{11}, \frac{5}{11}, \frac{8}{11}, \frac{11}{11}, \frac{13}{11}) \\
\text{segmentation} & (x_{a1}, x_{a2}, x_{b1}, x_{b2}, x_{c1}, x_{c2}) = (0, \frac{1}{6}, \frac{1}{6}, r, \frac{3}{6}, \frac{3}{6}, r, \frac{5}{6}, \frac{5}{6}, r, \frac{5}{6}, r) \\
\text{quasi-interlacing} & (x_{a1}, x_{b1}, x_{c1}, x_{a2}, x_{b2}, x_{c2}) = (0, r_1, \frac{1}{2}, \frac{1}{2}, r_1, \frac{1}{2} + 2r_1) \\
\text{quasi-partial segmentation} & (x_{a1}, x_{b1}, x_{c1}, x_{c2}, x_{a2}, x_{b2}) = (r_2, r_3, r_4, 1 - r_4, 1 - r_3, 1 - r_2)
\end{array}
\right.
\]

where \( r = (11 - \sqrt{73})/36 \approx 0.068, r_1 \approx 0.182, r_2 \approx 0.120, r_3 \approx 0.313, \text{ and } r_4 \approx 0.462. \) In the following sections, the first three candidates are shown to be SPNE, whereas the last two are not.

**Interlacing**

Because this is a perfectly symmetric configuration, it is obvious that all the first-order conditions in location competition are met. In order to show SPNE, it is sufficient to show that \((x_{b1}, x_{b2}) = (1/6, 4/6)\) is the maximizer of \(\pi_b(x_{b1}, x_{b2})\) given \((x_{a1}, x_{c1}, x_{a2}, x_{c2}) = (0, 2/6, 3/6, 5/6)\) due to symmetry of location.

We first show that \((x_{b1}, x_{b2}) = (1/6, 4/6)\) is the maximizer of \(\pi_b(x_{b1}, x_{b2})\) in the intervals of \(x_{b1} \in [0, 2/6]\) and \(x_{b2} \in [3/6, 5/6]\) for the interlacing configuration. The first-order conditions in the first-stage location equilibrium are given by the simultaneous equations \(\pi_b(x_{b1}, x_{b2})/\partial x_{b1} = \pi_b(x_{b1}, x_{b2})/\partial x_{b2} = 0\), which can be solved by using the Buchberger’s algorithm (Cox, Little and O’Shea, 1997). One of the Gröbner bases is shown to be a 101th-order single-variable polynomial of \(x_{a2}\) with integer coefficients. Computing all the solutions by the Jenkins-Traub algorithm, we can verify that there are three solutions are in the interval of \([3/6, 5/6]\). Among them, \(x_{b2} = 4/6\) is shown to be the unique solution that satisfies the first-order conditions. Plugging it into \(\pi_b(x_{b1}, x_{b2})/\partial x_{b1} = 0\) yields \(x_{b1} = 1/6\). Furthermore, we can verify that the second-order conditions for local maximum are satisfied. Hence, \((x_{b1}, x_{b2}) = (1/6, 4/6)\) is the maximizer of \(\pi_b(x_{b1}, x_{b2})\) given that the spatial arrangement is interlacing.

We next show that non-interlacing configurations \((x_{b1} \notin [0, 2/6] \text{ or } x_{b2} \notin [3/6, 5/6])\) cannot be an SPNE given \((x_{a1}, x_{c1}, x_{a2}, x_{c2}) = (0, 2/6, 3/6, 5/6)\). When \(b_2\) deviates its location, there are two different spatial arrangements up to permutation: \(a_1b_1b_2c_1a_2c_2\) and \(a_1b_1c_1b_2a_2c_2\). Note that the first arrangement involves a single-outlet firm \(b\) when
Applying the Buchberger’s algorithm again for these arrangements, we can show that neither spatial arrangement yields the profit higher than \( \bar{\pi}_b(1/6, 4/6) = t/108 \).

Putting these results together, we arrive at \((x_{b1}, x_{b2}) = (1/6, 4/6)\) as the global maximizer of \( \bar{\pi}_b(x_{b1}, x_{b2}) \). Because of location symmetry, the same is true for firms \( a \) and \( c \). Hence, the interlacing configuration

\[
(x_{a1}, x_{b1}, x_{c1}, x_{a2}, x_{b2}, x_{c2}) = (1/6, 2/6, 3/6, 4/6, 5/6, 6/6)
\]

is an SPNE.

Partial segmentation

The first-order conditions of the location competition are readily confirmed. Because outlet locations of firms \( b \) and \( c \) are symmetric, we only show that \((x_{a1}, x_{a2}) = (0, 8/16)\) is the maximizer of \( \bar{\pi}_a(x_{a1}, x_{a2}) \) in the intervals of \( x_{a1} \in [0, 3/16] \cup [3/16, 1] \) and \( x_{a2} \in [5/16, 11/16] \) given \((x_{b1}, x_{b2}, x_{c1}, x_{c2}) = (3/16, 5/16, 11/16, 13/16)\) and that \((x_{b1}, x_{b2}) = (3/16, 5/16)\) is the maximizer of \( \bar{\pi}_b(x_{b1}, x_{b2}) \) for all \( 0 \leq x_{a1} \leq x_{a2} \leq 8/16 \) given \((x_{a1}, x_{a2}, x_{c1}, x_{c2}) = (0, 8/16, 11/16, 13/16)\).

Applying the Buchberger’s algorithm to \( \partial \bar{\pi}_a(x_{a1}, x_{a2}) / \partial x_{a1} = \partial \bar{\pi}_a(x_{a1}, x_{a2}) / \partial x_{a2} = 0 \), one of the Gröbner bases is shown to be a 45th-order single-variable polynomial of \( x_{a2} \). Using the Jenkins-Traub algorithm, there is a unique solution \( x_{a2} = 8/16 \) that satisfies the first-order conditions. Plugging it into another Gröbner basis yields \( x_{a1} = 0/16 \). Furthermore, we can verify that the second-order conditions for local maximum are met. Hence, \((x_{a1}, x_{a2}) = (0/16, 8/16)\) is the maximizer of \( \bar{\pi}_a(x_{a1}, x_{a2}) \) under the partial segmentation arrangement. Similarly, \((x_{b1}, x_{b2}) = (3/16, 5/16)\) is shown to be the unique solution of \( \partial \bar{\pi}_b / \partial x_{b1} = \partial \bar{\pi}_b / \partial x_{b2} = 0 \) by computing a 37th-order single-variable polynomial of \( x_{b2} \). The second-order conditions for local maximum are satisfied. The same thing can be said for firm \( c \).

Next, we consider deviations to spatial arrangement that are not partial segmentation. First, given \((x_{b1}, x_{b2}, x_{c1}, x_{c2}) = (3/16, 5/16, 11/16, 13/16)\), there are two different arrangements up to permutation: \( a_1a_2b_1b_2c_1c_2 \) and \( a_1b_1a_2b_2c_1c_2 \). However, we can easily show that firm \( a \) cannot get the profit higher than \( \pi^*_a = 242t/12288 \) in either deviation. Second, given \((x_{a1}, x_{a2}, x_{c1}, x_{c2}) = (0, 8/16, 11/16, 13/16)\), there are two different arrangements up to permutation: \( a_1b_1a_2b_2c_1c_2 \) and \( a_1b_1a_2c_1b_2c_2 \). However, firm \( b \) cannot get the profit higher than \( \pi^*_b = 169t/12288 \) in either deviation. The same is true for firm \( c \).
Finally, consider the case that firm \( i = a, b, c \) chooses a single outlet. It is straightforward that none of the firms is able to obtain its profit higher than \((\pi_a^*, \pi_b^*, \pi_c^*) = (242t/12288, 169t/12288, 169t/12288)\).

Hence, the partial segmentation configuration

\[
(x_{a1}, x_{b1}, x_{b2}, x_{c1}, x_{c2}) = (0, 3/16, 5/16, 8/16, 11/16, 13/16)
\]

is an SPNE.

**Segmentation**

The first-order conditions of the location competition can be easily shown. Due to the location symmetry, it is sufficient to show that \((x_{a1}, x_{a2}) = (1/6 - r, 1/6 + r)\) is the maximizer of \(\tilde{\pi}_a\) for all segmentation configurations given \((x_{b1}, x_{b2}, x_{c2}, x_{c2}) = (3/6 - r, 3/6 + r, 5/6 - r, 5/6 + r)\).

Again applying the Buchberger’s algorithm to \(\partial \tilde{\pi}_a(x_{a1}, x_{a2})/\partial x_{a1} = \partial \tilde{\pi}_a(x_{a1}, x_{a2})/\partial x_{a2} = 0\), one of the Gröbner bases is shown to be a 37th-order single-variable polynomial of \(x_{a2}\). Using the Jenkins-Traub algorithm, there is a unique solution \(x_{a1} = 1/6 - r\). Moreover, we can easily show the second-order conditions for local maximum. Hence, \((x_{a1}, x_{a2}) = (1/6 - r, 1/6 + r)\) is the maximizer of \(\tilde{\pi}_a(x_{a1}, x_{a2})\) under the segmented arrangement. The same thing can be said for firms \(b\) and \(c\).

Thus, the segmentation configuration

\[
(x_{a1}, x_{a2}, x_{b1}, x_{b2}, x_{c1}, x_{c2}) = (1/6 - r, 1/6 + r, 3/6 - r, 3/6 + r, 5/6 - r, 5/6 + r)
\]

is an SPNE.

**Quasi-interlacing**

Calculating the first-order conditions of location equilibrium for arrangement \(a_1b_1c_1a_2b_2\) by the damped Newton’s method in Mathematica, we get \((x_{a1}, x_{b1}, x_{c1}, x_{a2}, x_{c2}, x_{b2}) = (0, r_1, 2r_1, \frac{1}{2}, \frac{1}{2} + r_1, \frac{1}{2} + 2r_1)\). However, this is not an SPNE because firm \(b\) can increase its profit by withdrawing its outlet \(b_2\).

**Quasi-partial segmentation**

Calculating the first-order conditions of location equilibrium for arrangement \(a_1b_1c_1a_2b_2\) by the damped Newton’s method in Mathematica, we have \((x_{a1}, x_{b1}, x_{c1}, x_{c2}, x_{a2}, x_{b2}) =\)
(r_2, r_3, r_4, 1 - r_4, 1 - r_3, 1 - r_2). However, this is not an SPNE because firm c can raise its profit upon relocating its first outlet from x_{c1} = r_4 to x_{c1} = 0.

**Appendix 3: Computations of the degree of mixing??**

Suppose there are five firms each can open a sufficiently large number n of outlets.

**Interlacing**

Firm \(i = 1, 2, \ldots, 5\) locates \(n\) outlets symmetrically at \(x_{ik} = k/n\) \((k = 1, 2, \ldots, n)\) up to rotation. The average distance from outlet 1 to all other outlets \(k\) of the same firm is

\[
d_s = \lim_{n \to \infty} \frac{1}{n-1} \sum_{k=2}^{n} \min\{|x_{i1} - x_{ik}|, 1 - |x_{i1} - x_{ik}|\} = \frac{1}{4}
\]

Because this is the same as the average distance to all other outlets of the different firms, \(d_d\), the degree of mixing in the interlacing configuration is \(d_s/d_d = 1\).

**Segmentation**

Assuming geographical symmetry, firm \(i\) establishes a continuum of firms in the interval of \([x_i, x_i + y_i]\), where \(x_i = i/5\) and \(y_i \in [0, 1/5]\) for \(i = 1, 2, \ldots, 5\). Let \(p^l_i\) and \(p^r_i\) be the prices of firm \(i\)’s outlets located at \(x = x_i, x_i + y_i\), respectively. In order to extract consumers’ surplus, the price of firm \(i\)’s outlet of at location \(z\) is given by

\[
p_i = \begin{cases} p^l_i + (x_i - z)^2 & \text{for } z \in [x_i, \tilde{x}_{i,i}] \\ p^r_i + (x_i + y_i - z)^2 & \text{for } z \in [\tilde{x}_{i,i}, x_i + y_i] \end{cases}
\]

and hence, the profit of firm \(i\) is defined as

\[
\pi_i = p^l_i (x_i - \tilde{x}_{i-1,i}) + \int_{x_i}^{\tilde{x}_{i,i}} p^l_i + (x_i - z)^2 \, dz + \int_{\tilde{x}_{i,i}}^{x_i+y_i} p^r_i + (x_i + y_i - z)^2 \, dz + p^r_i (\tilde{x}_{i,i+1} - x_i - y_i)
\]

where the market boundaries are

\[
\tilde{x}_{i,i} = \frac{p^l_i - p^l_{i-1}}{2y_i} + \frac{2x_i + y_i}{2} \\
\tilde{x}_{i-1,i} = \frac{p^l_i - p^l_{i-1}}{2(x_i - x_{i-1} - y_{i-1})} + \frac{x_i + x_{i-1} + y_{i-1}}{2}
\]

We solve the first-order conditions for prices, plug them into the profits, and then solve
the first-order conditions for locations:

\[
\frac{d \pi_i}{dx_i} = \frac{\partial \pi_i}{\partial x_i} + \sum_{j=1}^{5} \left( \frac{\partial \pi_i}{\partial p_{ij}} \frac{\partial p_{ij}}{\partial x_i} + \frac{\partial \pi_i}{\partial p_{ij}} \frac{\partial p_{ij}}{\partial x_i} \right) \\
= \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_i}{\partial p_{i-1}} \frac{\partial p_{i-1}}{\partial x_i} + \frac{\partial \pi_i}{\partial p_{i+1}} \frac{\partial p_{i+1}}{\partial x_i} = 0
\]

Imposing symmetry \( y_i = y \) for all \( i \), this condition is simplifies as

\[
625y^4 - 1875y^3 + 1325y^2 - 310y + 14 = 0
\]

which has a unique solution \( y^* \approx 0.059 \) in the interval of \( y \in [0, 1/5] \). Then, the average distances between two outlets belonging to the same firm and that belonging to the different firms are computed as

\[
d_s = \frac{1}{y^2} \int_0^y \int_0^y |x - z| \, dx \, dz = \frac{y}{3}
\]

\[
d_d = \frac{1}{2y^2} \int_0^y \left( \int_{1/5}^{1/5+y} |x - z| \, dx + \int_{2/5}^{2/5+y} |x - z| \, dx \right) \, dz = \frac{3}{10}
\]

Thus, the degree of mixing in the segmented configuration is \( d_s/d_d \approx 0.065 \).

References


Figure 1: Three SPNE of three multi-outlet firms
Figure 2: Convenience stores near Shinjuku station (light green: Seven-Eleven, blue: Lawson, black: FamilyMart, brown: am/pm, red: CircleKSunkus, yellow: unused in the analysis)