Self-organizing Marketplaces*

Takatoshi Tabuchi†

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Abstract

Dynamics of retail firms in marketplaces are analyzed, assuming that firms compete under monopolistic competition within a marketplace as well as between marketplaces and consumers are uniformly distributed over space. The number, size, and location of marketplaces or edge cities are analytically obtained. Furthermore, extending the model to a two-dimensional space, Christaller-Lösch system of hexagonal market areas is analytically derived.

Keywords: edge cities, agglomeration, Christaller-Lösch system, spatial competition, monopolistic competition.


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†Faculty of Economics, University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan. Fax: +81-3-5841-5521. E-mail ttabuchi@e.u-tokyo.ac.jp
1 Introduction

Ever since the seminal work of Hotelling (1929), spatial competition has been extended in a number of ways within the framework of oligopoly. When firms compete in location and the price of a homogeneous good, Hotelling (1929) conjectured that they agglomerate at the market center in order to obtain a larger market area. However, this is proved to be false by d’Aspremont et al. (1979), who showed that firms always locate apart in order to relax price competition. Otherwise, they are involved in Bertrand price competition, which pushes their profits down to zero.\footnote{The same can be said when firms compete in different strategies. For example, Peng and Tabuchi (2007) show that firms never locate back to back when they compete in location and variety.} In brief, the main message in spatial competition is that keen competition always leads to dispersion of firms over space.

It is true that some retail firms such as gas stations and convenience stores tend to locate apart, but it is also true that they often form clusters. Casual empiricism suggests that shopping centers and malls are prevalent and have been increasing in size and number everywhere in recent years. One could think of Broadway or Champs d’Elysees, where hundreds of shops and restaurants provide a wide array of differentiated goods and services.

Such a stylized fact of agglomeration of retail firms is in sharp contrast to the results obtained in the above literature on spatial competition. One of the crucial reasons for the contrast is substitutability of goods. If goods are homogeneous, it is no doubt that firms avoid Bertrand price competition in spite of the attractiveness of the market center. However, if the goods sold by firms are heterogenous, such competition would be relaxed. It is possible that the repulsion due to the fierce price competition may be outweighed by the attractiveness of the center in the case of heterogeneous goods.

As a matter of fact, agglomeration of retail firms is shown in the literature to be a Nash equilibrium by introducing heterogeneity of goods. de Palma et al. (1985) first showed...
that agglomerated configuration at the market center is a Nash equilibrium (which is the so-called the principle of minimum differentiation in the theory of product differentiation) when goods are sufficiently differentiated and/or the transport costs are sufficiently low. De Fraja and Norman (1993) showed the same result in the case of duopoly with the linear demand under several pricing schemes. Ago (2008) and Henkel et al. (2000) also showed the same result in the case of monopolistic competition.

Although firms choose to agglomerate at the market center under sufficient differentiation and/or low transport costs, this is not the only possible equilibrium. In reality, firms often locate in the suburbs of large cities due to easy access by cars for the benefit of consumers as well as low land rent for firms. It is therefore more appropriate to consider the case that in large cities there are multiple marketplaces, where many firms enter freely and sell differentiated goods and services under a monopolistically competitive market.

Table 1 describes the declining shares of retail employment in the central district, along with the increasing share in the suburbs in the Tokyo Metropolitan Area (TMA) during the postwar period. Such a tendency is also similarly found in the value of retail sales during the study period. Two definitions of the center are taken into account: the smaller center is CBD1 consisting of four wards located around Tokyo Station, and the larger one is CBD2 comprising twenty-three wards that include CBD1. TMA is composed of the center and suburbs, which encompasses four prefectures with a population greater than 17 million in 1960 and 34 million in 2007. Table 1 evidently shows how retail employment is growing much faster in the suburbs than in the center of the TMA during the postwar period.

Based on the foregoing observations, I build on Henkel et al. (2000). Self-organizing marketplaces across space, firms compete in price and location in order to attract consumers under a monopolistically competitive market, where each firm has a negligible impact on other firms in terms of their price and location strategies in the markets. They compete not only within the marketplace in which they locate but also between market-
places. The competition within a marketplace is keener as the number of firms at the same marketplace increases. However, such an agglomeration is not necessarily undesirable for firms because it can attract more consumers relative to other marketplaces.

This paper differs from Henkel et al. (2000) in three respects. First, stability of equilibrium is defined by dynamics with a mass of firms rather than by a strong Nash equilibrium with a discrete number of firms. This is because each firm has no strategic impact on others under monopolistic competition, so that the dynamics with a mass of firms should be more appropriate than a strong Nash equilibrium in the case of monopolistically competitive markets.

Second, unlike Henkel et al. (2000), some consumers are unable to go to a marketplace when transport costs are high. This is because the income net of transport costs of consumers located at a distance from the marketplace may become negative as the geographical space gets sufficiently large. In this case, new marketplaces, which are often called edge cities, would emerge in peripheral areas when cities grow sufficiently large in size. It should be noted that in spite of the fact that edge cities are shown to be prevalent in the real world (Garreau, 1991; McMillen and Smith, 2003), few analytical models of edge cities and subcenters have been developed in the literatures to the best of my knowledge. Exceptions are Cavailhes et al. (2007); Fujita and Ogawa (1982); Fujita et al. (1997); and Henderson and Mitra (1996). However, they consider two edge cities at the most, where firms produce rather than sell goods.

Third, the space is extended from one-dimensional to two-dimensional in order to meet a more realistic urban structure. Interestingly it is shown that the two-dimensional extension yields Christaller’s (1933) and Lösch’s (1940) hexagonal systems of marketplaces.

\[\text{Note that the edge cities considered by Garreau (1991) are not the same as marketplaces in this paper in that employment is dispersed in edge cities in Garreau (1991) whereas it is assumed to be concentrated in the city center here. Put differently, the commuting pattern is multicentric in Garreau (1991), whereas it is monocentric here.}\]
as a market outcome. Note that Löschian polygonal systems are investigated by Eaton and Lipsey (1976). However, firms produce rather than sell goods in their model.

The rest of the paper is organized as follows. Section 2 sketches the model by Henkel et al. (2000) briefly. Section 3 characterizes the agglomerated and symmetric equilibria and their stability. Section 4 then studies an evolutionary process of urban structures and show how edge cities emerge successively. Section 5 extends it to the two-dimensional space and obtains the hexagonal configuration of marketplaces. Section 6 concludes.

2 The model

Consumers are uniformly distributed over space with the density normalized to 1. For a moment, they are assumed to be distributed on a line segment $x \in [-L/2, L/2]$, where $L$ is the mass of consumers. They have the same CES utility with respect to a continuum $n$ of varieties of a horizontally differentiated good:

$$U = \left[ \int_0^n q(v, x)^{\sigma-1} \, dv \right]^{\frac{\sigma}{\sigma-1}},$$

(1)

where $q(v, x)$ is the consumption of variety $v$ at location $x$ and $\sigma > 1$ is the elasticity of substitution between any two varieties. The income constraint is given by

$$y = \int_0^n p(v)q(v, x)dv + tx,$$

(2)

where $t$ is the unit transport cost for visiting a marketplace and $x$ is the distance to a marketplace. Assume for a moment that

$$y - tL > 0$$

(3)

so that visiting any marketplace is possible throughout the line segment.

Each consumer maximizes (1) with respect to $q(v, x)$ subject to the income constraint (2). The consumer demand for variety $v \in [0, n]$ is given by

$$q(v) = \frac{p(v)^{-\sigma}}{\int_0^n p(v')^{1-\sigma} \, dv'} (y - tx).$$

(4)
The profit of firm $v$ is:

$$\Pi(v) = \int_{x \in X} (p(v) - c) q(v, x) dx - f.$$ \hfill (5)

where $X$ is the set of consumers who purchase a variety from firm $v$ at a marketplace. Firm $v$ maximizes its profit (5) with respect to its mill price given demand (4). The equilibrium mill price is given by

$$p^*(v) = \frac{c\sigma}{\sigma - 1},$$

which turns out to be constant for any variety $v$ and for any location. Because each firm can be treated symmetrically, we drop $v$ hereafter.

Under free entry and exit of firms, the equilibrium profit should be equal to zero:

$$\Pi = \frac{1}{n\sigma} \int_{x \in X} (y - tx) dx - f = 0$$ \hfill (6)

so that the number of firms in a marketplace is given by

$$n^* = \frac{1}{f\sigma} \int_{x \in X} (y - tx) dx.$$ \hfill (7)

Then, the indirect utility of a consumer located at $x$ from a marketplace is

$$V = \frac{\sigma - 1}{c\sigma} (n^*)^{-\frac{1}{\sigma}} (y - tx).$$

When there are multiple marketplaces, consumers are assumed to visit only one that yields the highest utility and consume all varieties available at the marketplace.

3 Market equilibrium

Assume that there are $m$ marketplaces at locations $x_1, x_2, ..., x_m$, which are restricted to the consumer distribution $[-L/2, L/2]$ for simplicity. Following Ginsburgh et al. (1985), the spatial equilibrium is such that

$$\Pi_i \leq 0 \quad \text{and} \quad \Pi_i n_i = 0 \quad \forall x_i \in [-L/2, L/2],$$

where $\Pi_i$ is the profit of firm $i$, and $n_i$ is the number of firms in market $i$. The equilibrium mill price is given by

$$p^*_i = \frac{c\sigma}{\sigma - 1},$$

which turns out to be constant for any variety $v$ and for any location.
where \( \Pi_i \) is the firm’s profit located in marketplace \( i \). The profit \( \Pi_i \) is a function of the distribution of firms over the marketplaces \((n_1, n_2, ..., n_m)\), where \( n_i \) is the number of firms in marketplace \( i \). Firms do not enter marketplace \( i \) if the anticipated profit is negative.

In order to examine stability of equilibrium, one has to define dynamics of firm behavior. I assume the following dynamics

\[
\dot{n}_i = \Pi_i (n_1, n_2, ..., n_m),
\]

where the dot denotes the time derivative. Dynamics (8) implies that firms are more attracted to marketplaces having higher profits. The dynamics (8) is stable if any infinitesimal perturbations in the distribution of firms result in a movement back toward the equilibrium. This can be checked by computing the eigenvalues of Jacobian of the RHS in (8).

In what follows, I focus on the fully agglomerated equilibrium and the symmetric equilibrium of two marketplaces. Asymmetric equilibria are not analytically tractable.

### 3.1 Agglomerated equilibrium

Suppose there are two marketplaces at \( x_1 \) and \( x_2 \) with \(-L/2 \leq x_1 < x_2 \leq L/2\). Let \( \hat{x} \) be the market boundary, i.e., the location of the marginal consumer, who is indifferent toward visiting either of them. Equating the indirect utilities of visiting both marketplaces yields the market boundary:

\[
\hat{x} = \begin{cases} 
-L/2 & \text{if } x_{\text{int}} \leq x_1 \\
x_{\text{int}} & \text{if } x_1 < x_{\text{int}} < x_2 \\
L/2 & \text{if } x_{\text{int}} \geq x_2,
\end{cases}
\]

where

\[
x_{\text{int}} \equiv \frac{y(r - 1) + t(x_1r + x_2)}{t(r + 1)}
\]

is the interior market boundary and \( r \equiv (n_1/n_2)^{\frac{1}{\sigma - 1}} \).
I show that agglomerated configuration is a stable equilibrium as follows. If an infinitesimal mass of firms is located at \( x_2 \in [x_1, L/2] \) while a all remaining mass of firms is located at \( x_1 \), then \( r \) goes to infinity. From (9) and (3), one gets

\[
x_{\text{int}} = \frac{y + tx_1}{t} > L + x_1 > x_2.
\]

Hence, \( \bar{x} = L/2 \), implying that no consumers visit marketplace 2, and that \( \Pi_2 \) is necessarily equal to 0 for any small increase in \( n_2 \). In other words, the agglomerated configuration is always a stable equilibrium. This suggests the lock-in effect in the location of the marketplace.

In order to upset the agglomerated equilibrium, the nonnegligible number of firms should simultaneously move from \( x_1 \) to \( x_2 \). This is possible if a coalition is formed among the firms (Henkel et al., 2000), which is however not allowed in the above dynamics with a mass of firms under the monopolistically competitive market.\(^3\)

From (7), the number of firms in the agglomerated equilibrium is computed as

\[
n_1^* = \frac{1}{f\sigma} \left[ yL - t \left( x_1^2 + \frac{L^2}{4} \right) \right].
\]

This is maximized when the marketplace is located at the center of the line segment \( x_1 = 0 \). This is due to the elastic demand for the differentiated goods.

Stability of the agglomerated equilibrium is guaranteed because \( \partial \Pi_1 / \partial n_1 < 0 \) and \( \Pi_2 < 0 \) hold when they are evaluated at \( (n_1, n_2) = (n_1^*, 0) \). The foregoing argument may then be summarized as follows:

**Proposition 1** The agglomerated configuration is always a stable equilibrium irrespective of its location.\(^4\)

\(^3\)This is also possible if there were developers who coordinated deviations from the agglomerated equilibrium (Fujita et al., 1997; Henderson and Mitra, 1996).

\(^4\)This proposition is valid even though the location of the marketplace is outside the consumer distribution \([-L/2, L/2]\).
Assuming the linear demand, Ago (2008) and De Fraja and Norman (1993) show that the equilibrium location of the agglomerated marketplace is at the center under sufficiently low transport costs. However, this proposition shows that the agglomerated marketplace is not necessarily at the center in a more general setting of the nonnegligible transport costs and the nonlinear demand (4) derived from the CES utility.

3.2 Symmetric equilibrium

Next, consider the case of two marketplaces symmetrically located about the center of the line segment such that \(x_1 + x_2 = 0\). Making use of symmetry, the equilibrium number of firms, (7), in each marketplace is readily computed as

\[
n^*_1 = n^*_2 = \frac{1}{2f} \left[ yL - t \left( \left( \frac{L}{2} - x_2 \right)^2 + x_2^2 \right) \right].
\]

This is maximized when \(x_2 = L/4\), where the sum of the consumer demand is the largest. Again, this is attributed to the elastic demand for differentiated goods.

Checking the signs of Jacobian of the RHS of (8), one obtains the stability condition of the symmetric equilibrium as follows:

\[
y < \frac{t}{4} \left[ (\sigma - 1) L + 4x_2 + \sqrt{(\sigma - 1) \left[ (\sigma + 1) L^2 - 16 \left( L/2 - x_2 \right)^2 \right]} \right].
\]

Examining (10), one can say that the symmetric equilibrium is stable when goods are close substitutes (\(\sigma\) large), the transport costs are high (\(t\) large), the consumer demand is large (\(L\) large), and the marketplaces are located far apart (\(x_2\) large).

When goods are close substitutes, consumers do not care for product variety, and hence, the agglomeration force is weak. This is in agreement with the result in new economic geography (Krugman, 1991) as well as that in spatial competition under product heterogeneity (de Palma et al., 1985).

When the transport costs are high, competition between the marketplaces is softened because the market boundary is not sensitive to changes in the size of marketplaces (the
integral part in (6)). However, competition within a marketplace does matter (n in the denominator in (6)). Because the latter effect dominates the former, the symmetric equilibrium turns out to be stable. As before, this agrees with the result in new economic geography as well as that in spatial competition.

Finally, when the marketplaces are located at a distance, the demand at the market boundary is small due to elastic demand with respect to distance to be covered for shopping. Because competition between the marketplaces is localized only at the market boundary in this model, small demand at the market boundary implies weak competition, which ensures stability of the symmetric equilibrium.

4 Evolution of spatial structure

Thus far, the consumer demand has been spatially fixed. However, it is of interest to consider endogenous locations of consumers together with those of marketplaces in a growing city in the following way. Each consumer resides on a plot of land, the length of which is normalized to 1.\textsuperscript{6} In order to receive the fixed income $y$, each consumer has to commute to the central business district, which is located at $x = 0$ and is assumed to be spaceless (Alonso, 1964). Because commuting involves costs, consumers eventually locate on the interval $[-L/2, L/2]$ in equilibrium, where $L$ is the population size as given by the length of the line segment.

As we saw in the previous section, both the agglomerated and symmetric equilibria are stable in case inequality (10) is satisfied. When there are more marketplaces, multiple equilibria are likely to exist. In order to refine multiple equilibria, assume that the population is initially small and is steadily growing exogenously as in Fujita and Krugman (1995). Inequality (10) implies that configurations with multiple marketplaces are stable when $\frac{E}{A} < \frac{1}{2}$.

\textsuperscript{5}Arakawa (2006) shows that this is not necessarily true if consumers can visit both marketplaces.

\textsuperscript{6}I assume away the land rent for analytical simplicity.
unlikely to be a stable equilibrium for sufficiently small $L$. This suggests that the initial equilibrium with a sufficiently small city is the agglomerated configuration, which is unique. Once the agglomeration is formed, it is necessarily stable from Proposition 1. Although the location of the marketplace can be anywhere in the city, it should coincide with the location of the central business district $x = 0$, to which all consumers commute.

Such an agglomerated equilibrium continues insofar as condition (3) is met. More precisely, because the maximum distance between the marketplace and consumer locations is now $L/2$, condition (3) is replaced with

$$y - tL/2 > 0. \quad (11)$$

Stated differently, if the population size $L$ grows and exceeds $2y/t$, then condition (11) is violated. In this case, since some consumers are unable to visit a marketplace with distance greater than $y/t$, firms in this single marketplace no longer can serve all consumers in the linear city. Consequently, the agglomerated configuration is no more an equilibrium, and hence, new marketplaces would emerge at both edges of the city, $x = \pm L/2$. They may be called the subcenters or edge cities, whereas the initial marketplace is called the center. Note that the unit transport cost $t$ consists of the unit shopping trip cost $t_s$ and the unit commuting cost $t_c$, i.e., $t = t_s + t_c$ unlike the preceding sections.

Consumers living close to the city center would visit it for shopping as well as working on the way back from commuting. Consumers living in the suburbs may also do the same at the city center. However, they are more likely to go to a marketplace at the subcenter near their residence because shopping is often done on weekends.

The spatial structure of the right part of the city is illustrated in Figure 1 (the left part is its mirror image). Let $L_i/2$ be the location of subcenter $i$ and $L_i$ be the city size when the $i$-th subcenter is about to appear. The threshold of city size $L_i$ is determined as follows. When the city size is slightly smaller than $L_i$, consumers living at the left of subcenter $i$ would commute to the city center for work and initially visit subcenter $i - 1$.
for shopping. Since the former distance is \( L_i/2 \) and the latter is \((L_i - L_{i-1})/2\), condition (11) is rewritten by

\[
y - t_c L_i/2 - t_s (L_i - L_{i-1})/2 > 0, \quad i \geq 1,
\]

where \( L_0/2 = 0 \) is the location of the city center. This inequality can be replaced with an equality for consumers locating sufficiently close to subcenter \( i \). Solving this difference equation yields the thresholds of city size:

\[
L_i \equiv \frac{2y}{t_s} \sum_{j=1}^{i} \left( \frac{t_s}{t_s + t_c} \right)^j, \quad i \geq 1.
\]

Observe that the thresholds are increasing in income \( y \) and decreasing in the transport costs for commuting \( t_c \) and shopping \( t_s \). That is, subcenters are likely to emerge when the income is low and the transport costs for commuting and shopping are high. The emergence of subcenters in recent years may be ascribed to rising opportunity costs of time, which constitute a large share of the transport costs, as well as increasing population in large cities.

In the sequel, I consider several stages according to the change in the number of subcenters.

(i) The first stage with \( L \in ]0, L_1] \)

Due to the existence of the commuting cost to the center, the city develops from the center \( x = 0 \) to the right (and left) in Figure 1 gradually as population increases. As the city increases in size, consumers at the city edges entail more costs of shopping trip and commuting and their net income \( y - (t_s + t_c) L/2 \) reduces and finally equals zero when \( L = L_1 \).

As shown in the second paragraph of this section, agglomeration at the center should be a unique equilibrium configuration in the first stage. The equilibrium number of firms is computed as

\[
n^*_1 = \frac{yL (L_1 - L/2)}{f\sigma L_1}.
\]
Since the agglomerated equilibrium is always stable from Proposition 1, there is no room for emergence of subcenters in this stage. The indirect utility of a consumer living at location \( x \) is given by

\[
V = \frac{\sigma - 1}{c \sigma} (n^*)^{\frac{1}{\sigma - 1}} [y - (t_s + t_c) x].
\] (13)

The number \( n^* \) of varieties in (13) is increasing in \( L \in [0, L_1] \), but the average distance of \( x \) is decreasing in \( L \). It can be readily shown that as the city develops, the welfare of all consumers in the city increases for a small \( L \). However, as the city grows further, their welfare near the center \( x \approx 0 \) still increases, but may decrease near the city edges \( x \approx \pm L/2 \) due to high commuting costs. Therefore, the welfare on average initially increases, but may or may not decrease when the city increases in size. This is because the benefit from the product variety may or may not be dominated by the commuting costs. Because the welfare is not transferable across consumers, I do not go into the welfare analysis any further.

(ii) The second stage with \( L \in [L_1, L_2] \)

When \( L \) exceeds \( L_1 \), consumers living in the interval of \( [L_1, L] \) are unable to go to the city center for shopping because their net income is negative (condition (11) is satisfied). However, they can instead visit one of the two new subcenters that emerge at \( x = \pm L_1/2 \). Since they never visit the center, firms located at the center cannot take over the whole demand. Put differently, firms located at the subcenters always have a positive demand by consumers at \( [L_1, L] \). This implies that even if the marketplaces at the subcenters are very small, they are always protected from the large marketplace at the center.

When \( L \) is not much larger than \( L_1 \), all consumers residing in \( ]-L_1/2, L_1/2[ \) go to the center for shopping and the rest of consumers residing in \( [-L/2, -L_1/2] \) and \( [L_1/2, L/2] \) visit the nearest subcenter. However, as \( L \) gets larger, the number of firms at the subcenters increases, and hence, the market boundaries between the center and subcenters would move inside the interval of \( ]-L_1/2, L_1/2[ \). As a result, although the two sub-
centers become smaller than the center when \( L \) is close to \( L_1 \), they may become larger when \( L \) approaches \( L_2 \). However, it can be shown that the marketplace in the center still survives in equilibrium.\(^7\)

Finally, when \( L \) becomes equal to \( L_2 \), the net income of a consumer located at the city edges is equal to zero, which leads to the emergence of additional subcenters at \( x = \pm L_2/2 \).

(iii) The \( i \)-th stage with \( L \in [L_{i-1}, L_i] \)

The evolutionary process (ii) is repeated for each stage. That is, there are \( 2i - 1 \) marketplaces at the center and subcenters in the \( i \)-th stage. The interval between subcenters \( i - 1 \) and \( i \) is computed as

\[
\frac{2y}{t_s} \left( \frac{t_s}{t_s + t_c} \right)^i.
\]

Because the location of each subcenter is defined by the sum of the above intervals, it is given by (12). Since sequence (14) is decreasing in \( i \), we obtain the following.

**Proposition 2** In the one-dimensional space, as the subcenters are farther away from the city center, their intervals get narrower.

This proposition suggests that the marketplaces are smaller in size depending on the distance from the center because their hinterlands get smaller. This is consistent with casual observations that the sizes of marketplaces in commuter towns and exurbs are small in size as they are far away from the city center. Note however that subcenters may become larger than the center as demonstrated in the above second stage. This may correspond to the prosperous shopping malls in the suburbs versus the stagnant central cities often observed in Japan’s small cities.

\(^7\)This is because the market boundary

\[
\tilde{x} = \frac{y}{t_s + t_c} \frac{(t_s + t_c) n_1^{1/(\sigma-1)} - t_c n_2^{1/(\sigma-1)}}{(t_s + t_c) n_1^{1/(\sigma-1)} - (t_s - t_c) n_2^{1/(\sigma-1)}}
\]

is shown to be in the interval of \( [0, L_1/2] = [0, y/(t_s + t_c)] \) for all relevant values of the parameters.
In order to gain further insight, I impose an assumption that the commuting cost \( t_c \) is sufficiently low but not zero hereafter. This assumption may be justified partly because shopping trips are more elastic than commuting trips with respect to distance costs, and partly because the commuting cost is reimbursed by companies in, say, Japan.\(^8\)

Setting \( t_c = 0 \) in (14), it can be easily verified that each interval between the neighboring marketplaces is equal. This implies that each marketplace is of equal size except for the two edges. Thus, we have the following.

**Proposition 3** In the one-dimensional space, when the commuting cost is sufficiently small, all the marketplaces are of the same size except for the two subcenters near the edges. The two edge marketplaces are smaller (resp. larger) if \( L \in ]L_{i-1}, (L_{i-1} + L_i)/2 [\) (resp. \( L \in ](L_{i-1} + L_i)/2, L_i)\]).

Because the intervals of marketplaces are identical, there is no locational difference between the marketplaces but for the two edge marketplaces. This is the first statement of the proposition. The second statement is that the two edge marketplaces can be larger or smaller in size than the others depending on the length of the hinterlands near the city edges. When \( L \in ]L_{i-1}, (L_{i-1} + L_i)/2 [,\) consumers outside the edge locations \( x = \pm L_{i-1}/2 \) are relatively few. Because of the locational disadvantage, the size of the edge marketplaces should be smaller than that of the others. This corresponds to the early development stages of edge cities. The opposite thing can be said when \( L \in ](L_{i-1}+L_i)/2, L_i)\]. Consumers outside the edge locations are relatively many, and therefore the edges have better access, i.e., locational advantage. This may correspond to large shopping centers and malls, which are often observed in the suburbs of large cities. Thus, the expansion of the hinterlands changes locational disadvantage to locational advantage according to development stages.

\(^8\)This assumption may be consistent with edge city employment (Garreau, 1991), which drastically reduces the commuting cost \( t_c \). However, allowing relocation of jobs in this model is analytically intractable as inferred from the analysis in Fujita and Ogawa (1982).
5 Two-dimensional extension

So far, the analysis has been confined to the one-dimensional space. This is extended to two-dimensions in this section because the geographical space in the real world is better approximated by a two-dimensional space, while keeping the assumption that the commuting cost \( t_c \) is sufficiently low but not zero.

Consider a featureless plane with a city center, where consumers are uniformly distributed around the city center over the two-dimensional disc. As the population \( L \) grows continuously, they are spreading around the city center and forming a disc. The first stage is not very different from the one-dimensional case. The agglomerated configuration continues to be a stable equilibrium for all \( L \in ]0, L_1] \), where \( L_1 \equiv \pi y^2/t_s^2 \). The equilibrium number of firms is

\[
n^*_1 = \frac{yL \left( 3\sqrt{L_1} - 2\sqrt{L} \right)}{3f\sigma\sqrt{L_1}},
\]

which is qualitatively similar to that in the one-dimensional case.

When the city size slightly exceeds \( L_1 \), consumers inside a circle with a radius of \( y/t_s \) (i.e., the shaded disc in Figure 2) go to the center, but those outside the circle cannot because the income net of the transport costs becomes negative. This suggests that there is a continuum of equilibrium location candidates for edge cities on the thin ring (i.e., the area outside the shaded disc and inside the dashed circle) in the two-dimensional space. On the other hand, they are confined to the two location points \( x = \pm L_1/2 \) in the case of the one-dimensional space.

Suppose three edge marketplaces emerge on the circumference of the shaded disc symmetrically as marked by \( I_a \), \( I_b \) and \( I_c \) in Figure 2 when \( L \) reaches \( L_1 \). Then, drawing the three circles with the centers at \( I_a \), \( I_b \) and \( I_c \) and the same radius \( y/t_s \) in Figure 2, one can readily verify that they exactly serve all consumers located on the thin ring. Edge marketplaces \( I_a \), \( I_b \) and \( I_c \) serve the one third of the thin ring from \( I'a \) to \( I'b \), \( I'b \) to \( I'c \) and \( I'c \) to \( I'a \), respectively, and the central marketplace serves the shaded
disc. No competition takes place between any pair of the marketplaces because none of
the four market areas overlaps each other due to the following reasons. First, since the
distance between any pair of the edge marketplaces is two times $y/t_s$ when $L = L_1$, their
marketplaces are adjoining but are not overlapping. Second, the relative number of firms
between the central marketplace and one of the edge marketplaces goes to infinity when
$L = L_1$. Using this fact together with (9), one can easily show that the market boundary
between them cannot be interior. Because consumers outside the shaded disc are unable
to visit the central marketplace for shopping, it must be that the market boundary of the
central marketplace is given by the circle with radius $y/t_s$. This means that the small
dge marketplaces are protected. Hence, this configuration is necessarily stable for any
strong perturbations.

The least number of edge marketplaces that can serve the entire consumers when
$L = L_1$ is three. What if more than three edge marketplaces emerge simultaneously on the
circumference of the shaded disc when $L = L_1$? Then, some of the market areas overlap,
which may cause instability of the equilibrium configuration by strong perturbations, say,
coalition building in Henkel et al. (2000). As the number of edge marketplaces increases,
such overlaps tend to increase. It thus follows that the equilibrium configuration with
three edge marketplaces would survive in the long-term evolutionary process as compared
to any equilibrium configurations with more than three edge marketplaces. For these
reasons, I set the three edge marketplaces at $I_a$, $I_b$ and $I_c$ when $L$ reaches $L_1$.

When $L$ exceeds $L_1$ further, but still slightly, consumers located near the three $I'$ in
Figure 2 are unserved. Therefore, three more edge marketplaces are to be immediately
established at the three locations $I'$. In sum, when $L$ slightly exceeds $L_1$, there are one
center and six subcenters $I+I'$ equidistantly located around the circumference of a circle
with radius $y/t_s$, thus constituting a regular hexagon.

As the population of the city keeps growing, consumers are continuously spreading
around the city center with one central marketplace and six edge marketplaces $I+I'$
while $L \in [L_1, 3L_1]$. When $L$ slightly exceeds $3L_1$, exactly six location points are not served. They are marked by II in Figure 3, which is an extended version of Figure 2. Their locations are symmetric on the circumference of a circle with radius $\sqrt{3y/t_s}$. Thus, adding the first group of six marketplaces I+I′ to the second group of six new marketplaces II, there are thirteen marketplaces for $L \in [3L_1, 4L_1]$, and then there are nineteen marketplaces with an addition of the third group of six new marketplaces III as illustrated in Figure 3.

Continuing these processes of the emergence of new edge cities, one can obtain the equilibrium configuration with regular hexagonal market areas, which is a two-dimensional extension of Proposition 3.

**Proposition 4** In the two-dimensional space, Christaller-Lösch’s hexagonal configuration is self-organized in equilibrium.

A sketch of the proof is summarized as follows. Given the location of the center, place subcenters around the center with the equally-spaced interval of $y/t_s$ between the nearest subcenters so that each market area of each subcenter constitutes the same regular hexagon. Because of the nature of regular hexagons, the location of each subcenter or the center is always an intersecting point of six circles of the nearest subcenters or the center as depicted in Figure 3. This implies that whenever the population $L$ slightly exceeds the threshold, consumers located just outside some of intersecting points on the circumference of a disc are unserved, and hence, their locations must coincide with the locations of emerging subcenters as intersecting points of six circles with a radius of $y/t_s$. This statement is always true because the disc over which consumers are uniformly distributed is expanding continuously with radius $\sqrt{L/\pi}$ for growing population $L$, and because any subcenters are located symmetrically around one of circles with the center.

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9Although the equilibrium of the entire hexagonal configuration is obtained, its stability conditions cannot be derived. This is because the integrals of elastic demand along the circumference of the disc and over the hexagons, and hence Jacobian of the RHS of (8), cannot be obtained analytically.
The hexagonal configuration depicted in Figure 3 is self-organized without any presence of social planners. It is similar to Figure 6 in chapter B of part I of Christaller (1933) or Figure 24 in chapter 10 of Lösch (1940). However, it differs from Christaller (1933) in that the hexagons are not nested because there is only one good here. It is expected that Christaller’s hierarchical system of nested hexagons is self-organized by introducing multiple goods having different parameters.\textsuperscript{10} For example, goods with high transport costs ($t_s$ large) form marketplaces with short intervals, whereas those with low transport costs ($t_s$ small) form marketplaces with long intervals. As a result, large marketplaces with long intervals would emerge, offering a wide array of goods and small marketplaces with short intervals offering few goods.

Finally, when the commuting cost $t_c$ is not negligible, the equilibrium configuration is expected to be a two-dimensional version of Figure 1. Namely, it would consist of successive hexagons, but the sizes of hexagons would gradually shrink according to the distance from the center. This would be also true when the population density is decreasing in the distance from the center.

\section{Conclusion}

I have extended the model of Henkel et al. (2000), where firms compete not only within a marketplace, but also between marketplaces in order to uncover the number, size, and locations of marketplaces that constitute central and edge cities. Empirical evidence in Japan’s large cities shows that the retail share in the suburban areas has been rising as compared to that in the central areas in these years. The results in this paper agree with the evidence. Furthermore, this paper has shown that Christaller-Lösch’s hexagonal configuration self-organizes endogenously in the monopolistically competitive retail market.\textsuperscript{10} To be more precise, multipurpose shopping should be carried out by consumers in order to self-organize the nested hexagonal configuration (Hsu and Holmes, 2009; Quinzii and Thisse, 1990).
Thus, it may be safely concluded that the model in this paper is able to describe the real world as well as Christaller-Lösch’s ideal world.

In this paper, the land rent is beyond the scope of this paper although it is of importance in urban economic theory (Alonso, 1964). If land rent is included in this model, the spatial difference in the transport costs, \( t_s \) and \( t_c \), are compensated by the land rent for all consumers visiting the same marketplace. That is, the income net of the transport costs and land rent is equal for all consumers going to the same marketplace. However, the net income is different between consumers visiting different marketplaces because the surplus accruing from the product variety differs between marketplaces. It can be shown from a preliminary analysis that adding land rent to the model does not alter the overall results much.

I have chosen to focus on the population increase as an exogenous driving force of emerging subcenters. The population increase accompanies not only suburbanization of residential areas, but also job decentralization. The latter is getting common in recent U.S. metropolitan areas (Lee, 2007), but is not taken into account in this paper. If there are workplaces in the suburbs, the results would not change in the case of sufficiently small commuting cost, which is assumed in Proposition 3 and section 5. The next line of research to be addressed may be incorporating endogenous job decentralization together with the nonnegligible commuting cost and land rent a la Fujita and Ogawa (1982), although such an extension will not be an easy task.

References


[23] Peng, S.-K., Tabuchi, T., 2007. Spatial competition in variety and number of stores, 

58, 1101-1119.
Table 1: Employment in retail industry in Tokyo Metropolitan Area

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Notes:
CBD1 consists of 4 wards of Chiyoda, Chuo, Minato and Shinjuku.
CBD2 consists of 23 wards including CBD1.
TMA consists of 4 prefectures of Tokyo, Kanagawa, Chiba and Saitama.
Figure 1: Spatial structure of a city

Figure 2: Locations of edge cities when $L$ slightly exceeds $L_1$
Figure 3: Christaller’s hexagonal configuration with 1 center, 6 subcenter I, 6 subcenter II, and 6 subcenter III