Changes in transport and non-transport costs: local vs global impacts in a spatial network

Kristian Behrens∗ Andrea R. Lamorgese†
Gianmarco I.P. Ottaviano‡ Takatoshi Tabuchi§

April 10, 2007 (final version)

Abstract

We develop a multi-country Dixit-Stiglitz trade model and analyze how industry location and welfare respond to changes in: (i) transport frictions (e.g., infrastructure, transportation technology); and (ii) non-transport frictions (e.g., tariffs, standards and regulations). We show that changes in non-transport frictions, which are usually origin-destination specific, do not allow for any clear prediction as to changes in industry location and welfare; whereas changes in transport frictions, which are usually not origin-destination specific, may allow for such predictions. In particular, we show that reductions in transport frictions occurring at links around which the spatial network is locally a tree are Pareto welfare improving.

Keywords: trade frictions; multi-country trade models; monopolistic competition; spatial networks; international integration

JEL Classification: D58; F12; F17; R12

∗Corresponding author: CORE, Université catholique de Louvain, 34 voie du Roman Pays, 1348 Louvain-la-Neuve, Belgium. Office: +3210474305. E-mail address: behrens@core.ucl.ac.be
†Research Department, Bank of Italy, Roma.
‡Università di Bologna, Italy; Bruegel, FEEM, and CEPR.
§Faculty of Economics, University of Tokyo, Japan.
1 Introduction

Even in the age of globalization international trade is hampered by trade costs. These arise from multiple sources and, although they have decreased markedly over the last half century, are still far from negligible. Anderson and van Wincoop (2004, pp. 692-693), for example, estimate that the tax equivalent of trade costs for industrialized countries amounts to 170%, which roughly breaks down into 21% transportation costs, 44% border related trade barriers, and 55% local retail and distribution costs. In all three respects, these costs are even larger in the case of less developed countries.

Trade costs are central to trade theory in so far as it focuses on the positive and normative effects of trade liberalization. Standard models, however, generally fail to acknowledge the possible different impacts of different types of trade costs. In particular, as argued by Deardorff (1984, p.470), the specificities of transport costs have usually been almost completely ignored:

“[..] like frictions in physics, transport costs are almost universally ignored in trade models in the sanguine hope that if included they would not materially affect the results.”

Three reasons explain this neglect. First, trade theory has focused almost universally on the most ‘visible’ costs of trading goods internationally, namely tariff and tariff-equivalent barriers. These offer an attractive subject to study, because they are determined by trade policy and are, therefore, endogenous to the process of economic decision making. In fact governments can change them instantaneously. Partly as a result, detailed tariff data are available for most countries, which makes empirical investigation relatively easy. This contrasts starkly with the time and resources required to affect transport costs as well as with the sparse availability of the corresponding data (Anderson and van Wincoop, 2004). Second, often causing problems of equilibrium indeterminacy, transport costs are difficult

\footnote{Empirical works usually disaggregate transport and trade costs in order to assess their relative influence on the structure and evolution of world trade (see, e.g., Baier and Bergstrand, 2001). In this respect, theory lags behind empirics.}
to include in the paradigm of perfect competition that represents the backbone of trade theory (Falvey, 1976; Cassing; 1978). Third, when it comes to imperfectly competitive models, there is a widespread belief that the different components of trade costs can be reduced without loss of generality to a single parameter. Samuelson’s (1954) *iceberg approach*, later used in most of trade theory and economic geography, provides a neat illustration of how a unique parameter subsumes all the impediments to trade, including tariff barriers and transport costs (Helpman and Krugman, 1985; Fujita et al., 1999). The underlying assumption is that all the components of trade costs affect industry location and trade patterns in the same way.

The ‘equivalence assumption’ of tariffs and transport costs is problematic for several reasons. Most naturally, tariffs generate revenues whereas transport costs do not, so their impact on welfare is bound to be generally different. Yet, there are more subtle issues that are not tied to considerations of tariff proceeds. Behrens et al. (forthcoming) have recently shown that international trade costs and national transport costs have quite different impacts on the location choices of mobile firms, since they affect prices and profits differently. In this paper, we develop these results further and show that changes in international transport costs and changes in border related trade frictions (e.g., tariffs, standards and regulations) usually have different impacts on the location of firms, the structure of trade, and the welfare of nations. The main reason we highlight in the present paper is that transportation must occur along some given *transport routes* and is subject to international arbitrage by profit maximizing agents. While arbitrage also partly occurs for tariffs, since large firms may go multinational in order to jump tariff barriers, the impact of tariffs on the location of exporting firms is more complicated to analyze as ‘routes’ do not generally exist in this case.

Building on the *M*-country version of Krugman’s (1980) model, as developed by Behrens et al. (2005), we investigate the positive and the normative impacts of deeper international integration. To do so, we use an approach that blends aspects of new trade theory and transport economics. In particular, while following the iceberg approach, we use a more realistic description of geography by assuming that the transportation in-
Infrastructure between countries is represented by a network along which shipping must occur. Stated differently, countries are no longer floating islands in some abstract space, but are now characterized by their address in the transportation network. Such an approach allows us to break down trade costs into a transport component and a non-transport component. Shipping goods between any two countries in the network then incurs both: (i) transport costs, which are associated with the shortest path between the two countries; and (ii) non-transport costs, such as tariffs and non-tariff border barriers, which are modeled as country-pair specific \textit{ad valorem} barriers in the usual way. Our approach naturally leads to a graph representation of the space economy. This is a major departure from existing multi-country trade models (e.g., Behrens et al., 2005), because it explicitly takes into account the fact that shipping occurs along particular routes which have a concrete structure in the geographical space.

Our analysis reveals that changes in transport and non-transport costs have quite different effects on the location of firms, the structure of trade, and the welfare of nations. On the one hand, non-transport costs generally have a \textit{global impact} as changes in one country’s barriers induce feedbacks that affect the location of firms in the global economy. Formally, the bilateral trade cost matrix in the $M$-country case consists of $M \times M$ independent parameters. Thus, even in the simplest case with just three countries very limited general results are available. On the other hand, changes in transport costs generally have only a \textit{local impact} as feedbacks have a steep distance decay. The intuition behind these results is that non-transport costs work as if there were direct links between any pair of countries; whereas transportation occurs along specific routes so that many pairs of countries do not have direct links (and a ‘triangle inequality’ naturally applies to indirect links). Formally, the bilateral transport cost matrix in the $M$-country case consists of less than $M \times M$ independent parameters and has a special structure that derives from the properties of the transportation network and the associated network metric. It is precisely the interposition of other countries that weakens the feedbacks between non-adjacent countries and makes the effects of changing transport costs localized. Although general results for an arbitrary transport network do not exist, we show that clear-cut
results can be derived for transportation improvements in parts of the network that have locally a tree-structure. In particular, any decrease in transport costs constitutes in this case a Pareto improvement in the global economy.

The remainder of the paper is organized as follows. Section 2 presents the model and Section 3 lays out the spatial structure of the economy. Sections 4 and 5 investigate the impacts of an increase in the freeness of trade. In these two sections, to disentangle the various effects and obtain clear-cut results, we distinguish between reductions in non-transport costs and reductions in transport costs, respectively. We show that, when the transportation network can be locally described by a tree, the latter case allows for clear predictions whereas again nothing can be said in general in the former case. Section 6 concludes and suggests future research directions.

2 The model

Our model is a multi-country extension of Krugman’s (1980) model. Consider a world with $M > 2$ countries, subscripted by $i = 1, 2, \ldots, M$. Each country is endowed with an exogenously given mass of $L_i$ workers-consumers, each supplying inelastically one unit of labor. Hence, both the world population and the world labor endowment are fixed at $L \equiv \sum_i L_i$. Labor is the only production factor and it is internationally immobile.

2.1 Preferences

Consumer preferences of a representative agent in country $j$ are defined over a homogeneous good and a continuum of varieties of a horizontally differentiated good:

$$U_j = D_j^\mu H_j^{1-\mu}$$

with $0 < \mu < 1$ and

$$D_j = \left[ \sum_i \int_{\Omega_i} d_{ij}(\omega)^{(\sigma-1)/\sigma} \, d\omega \right]^{\frac{\sigma}{\sigma-1}}.$$

\[^2\text{We alleviate notation by dropping summation ranges when there is no possible confusion.}\]
In the above expressions, $H_j$ stands for the consumption of the homogeneous good and $D_j$ for the (aggregate) consumption of the differentiated good; $d_{ij}(\omega)$ for the consumption of variety $\omega$ when it is produced in country $i$; and $\Omega_i$ for the set of varieties produced in that country. The parameter $\sigma > 1$ measures both the own- and cross-price elasticities of demand for any variety of the differentiated good.

Let $p_H$ be the price of the homogenous good. Maximization of utility (1) under the budget constraint

$$p_H H_j + \sum_i \int_{\Omega_i} p_{ij}(\omega)d_{ij}(\omega)d\omega = w_j,$$

yields the following individual demand in country $j$ for variety $\omega$ produced in country $i$: $d_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} P_j^{1-\sigma} \mu w_j$,

where $p_{ij}(\omega)$ is the delivered price of variety $\omega$; $w_j$ is the wage rate; and $P_j$ is the CES price index in country $j$, given by

$$P_j = \left( \sum_i \int_{\Omega_i} p_{ij}(\omega)^{1-\sigma}d\omega \right)^{\frac{1}{1-\sigma}}.$$

### 2.2 Technologies

The production of any variety of the differentiated good takes place under internal increasing returns to scale by a set of monopolistically competitive firms. This set is endogenously determined in equilibrium by free entry and exit. We denote by $n_i$ the mass of firms located in country $i$, and by $N \equiv \sum_i n_i$ the total mass of firms in the world economy. Production of any variety of the differentiated good requires a fixed and a constant marginal labor requirement, labeled $F$ and $c$ respectively. Increasing returns to scale, costless product differentiation, and the absence of scope economies yield a one-to-one equilibrium relationship between firms and varieties, so we will use the two terms interchangeably from now on.

In what follows, we assume that the trade frictions for the differentiated good are of the *iceberg form*: for one unit of any variety to arrive in country $j$, when shipped from country $i$, $\tau_{ij} > 1$ units have to be dispatched from the country of origin. Hence, a firm
in country \( i \) has to produce \( x_{ij}(\omega) \equiv L_j d_{ij}(\omega) \tau_{ij} \) units to satisfy final demand \( L_j d_{ij}(\omega) \) in country \( j \). We also assume that trade frictions are symmetric regardless of the direction of trade, i.e., \( \tau_{ij} = \tau_{ji} \). Though restrictive, especially when it comes to empirical work, this assumption is useful for deriving further insights in the general multi-country case.

The symmetry of technologies across firms and countries implies that, in equilibrium, firms differ only by the country they are located in. Accordingly, we may simplify notation by dropping the variety label \( \omega \) from now on.

Using (4), each firm in country \( i \) maximizes profit

\[
\Pi_i = \sum_j (p_{ij} - cw_i \tau_{ij}) L_j \frac{p_{ij}^{-\sigma}}{P_j^{1-\sigma}} \mu w_j - F w_i
\]

with respect to all its prices \( p_{ij} \), taking the price aggregates \( P_j \) and the wages \( w_j \) as given. This yields the equilibrium prices

\[
p_{ij} = \frac{\sigma}{\sigma - 1} cw_i \tau_{ij}.
\]

The production of the homogeneous good is, on the contrary, carried out by perfectly competitive firms under constant returns to scale. Without loss of generality, the unit labor requirement is normalized to one. Perfect competition implies pricing at marginal cost, which, given the normalization of the unit input coefficient, is equal to the wage. Finally, for reasons made precise below, we assume that the homogeneous good can be costlessly traded across all countries.

### 2.3 Market outcome

Free entry and exit in the differentiated industry implies that profits are non-positive in equilibrium. This is the case in country \( i \) when the equilibrium operational scale satisfies

\[
x_i \equiv \sum_j L_j d_{ij} \tau_{ij} \leq \frac{F(\sigma - 1)}{c}, \tag{8}
\]

where \( x_i \equiv \sum_j x_{ij} \) is the firm’s total production inclusive of output lost in shipping. Inserting (4) and (5) into (8), multiplying both sides by \( p_{ij} > 0 \), and using (7), we get:

\[
\sum_j w_i^{-\sigma} w_j \phi_{ij} L_j \leq \frac{\sigma F}{\mu}, \quad i = 1, 2 \ldots, M, \tag{9}
\]
where $\phi_{ik} \equiv \tau_{ik}^{-\sigma}$ is a measure of trade freeness, valued one when trade is free (i.e., $\tau_{ik} = 1$) and limiting zero when trade is inhibited (i.e., $\tau_{ik} \rightarrow \infty$). If (9) holds as a strict inequality for country $j$, $n^*_j = 0$ in equilibrium since no firm can break even there, whereas $n^*_j \geq 0$ otherwise.

Multiplying both sides of (9) by the positive $n_i$ and summing across countries, we get $N = \mu L/F\sigma$: in equilibrium the world mass of firms is constant and proportional to world population. Using this result, it turns out to be convenient to rewrite condition (9) in terms of shares. In particular, after defining $\theta_i \equiv L_i/L$ and $\lambda_i \equiv n_i/N$, it can be expressed as follows:

$$RMP_i \equiv \sum_j w_i^{-\sigma} w_j \phi_{ij} \theta_j \leq 1, \quad i = 1, 2, \ldots, M,$$

(10)

where $RMP_i$ stands for the real market potential (henceforth, RMP) in country $i$, associated with the industry distribution $\lambda$ (see Head and Mayer, 2004).

As can be seen from expression (10), the RMP in each country generally depends on the whole distributions of industry ($\lambda$), the size distribution ($\theta$), and factor prices ($w$). The latter enters the model in a highly non-linear way and makes the analysis especially complicated. In what follows, we therefore restrict our analysis to the case in which at least some production of the homogeneous product takes place in all countries. When combined with costless trade in that good, this is a sufficient condition for factor price equalization (henceforth, FPE) to hold. Formally, factor price equalization, i.e. $w_i = 1$ for all $i = 1, 2, \ldots, M$, requires any $M - 1$ dimensional subset of countries to be unable to satisfy world demand for the homogeneous good (see, e.g., Baldwin et al., 2003). Let $\ell_i$ be the amount of labor employed by a representative firm in country $i$. For homogeneous good production to take place everywhere, the total mass of workers in each country must exceed the total labor requirement in the differentiated industry, i.e., $L_i > n_i \ell_i$ for all $i$. Therefore, since $L_i = \theta_i L$ and

$$n_i \ell_i = \lambda_i N \left( F + c \sum_j x_{ij} \right) = \lambda_i \frac{\mu L}{F\sigma} \left[ F + c \frac{F(\sigma - 1)}{c} \right] = \lambda_i \mu L,$$

the condition for factor price equalization to hold regardless of $\lambda$ reduces to

$$\theta_i > \mu$$

(11)
for all $i$. Put differently, the expenditure share $\mu$ on the differentiated good must be small enough for the homogeneous good to be produced everywhere. We assume, in what follows, that condition (11) holds for all countries such that $w_j = 1$ for all $j$. This makes the homogeneous good a natural choice for the numéraire and allows us to refer to $\theta$ as the expenditure distribution.

### 2.4 Spatial equilibrium

A spatial equilibrium is such that the RMP is equalized to one across all countries hosting a positive measure of firms, whereas it falls short of this value in countries devoid of firms in the differentiated sector. In other words, all firms are equally profitable in all countries in which they operate, and they make zero profits due to free entry and exit. Formally, the conditions for a spatial equilibrium are as follows:

$$
RMP_i = 1 \quad \text{if} \quad \lambda_i^* > 0, \\
RMP_i \leq 1 \quad \text{if} \quad \lambda_i^* = 0.
$$

(12)

The industry shares $\lambda_i$ are $M$ endogenous unknowns, whereas the expenditure shares $\theta_i > 0$, as well as the trade freeness measures $0 < \phi_{ij} < 1$, are exogenous parameters. We make notation more compact by recasting it in matrix form. Let

$$
\Phi \equiv \begin{pmatrix}
\phi_{11} & \phi_{12} & \cdots & \phi_{1M} \\
\phi_{21} & \phi_{22} & \cdots & \phi_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
\phi_{M1} & \phi_{M2} & \cdots & \phi_{MM}
\end{pmatrix}, \quad 
\lambda \equiv \begin{pmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_M
\end{pmatrix}, \quad \text{and} \quad
\theta \equiv \begin{pmatrix}
\theta_1 \\
\theta_2 \\
\vdots \\
\theta_M
\end{pmatrix},
$$

where $\lambda^T 1 = \theta^T 1 = 1$ by definition of shares (in what follows, $1$ stands for the $M$-dimensional vector whose components are all equal to one). Using these definitions, and letting diag($x$) stand for the diagonal matrix obtained from the vector $x$, the $M$ equilibrium conditions (12) can be expressed in matrix notation as follows:

$$
RMP = \Phi \text{diag}(\Phi \lambda)^{-1} \theta \leq 1,
$$

(13)

with complementary slackness $(RMP_i - 1)\lambda_i = 0$ for all $i$. 

9
In what follows, we restrict our analysis to the meaningful case of interior equilibria, i.e., equilibria in which every country hosts a strictly positive measure of monopolistically competitive firms. In this case, (13) holds with equality for all countries, i.e., the equilibrium industry distribution $\lambda^*$ satisfies the following condition:

$$\Phi \text{diag}(\Phi \lambda^*)^{-1}\theta = 1 \quad \text{or, alternatively,} \quad \theta = \text{diag}(\varphi)\Phi \lambda^*.$$ 

Letting $\varphi \equiv \Phi^{-1}1$, letting $|\Phi|$ denote the determinant of $\Phi$, and letting $f_{ij} (= f_{ji}$ by symmetry of $\Phi$) stand for the cofactor of $\phi_{ij}$, these conditions can be rewritten component by component as follows:

$$\theta_i = \varphi_i \sum_k \phi_{ik} \lambda_k^* \quad \text{with} \quad \varphi_i = \frac{\sum_j f_{ji}}{|\Phi|}. \quad (14)$$

Two remarks are in order. First, countries can be thought of as nodes in the spatial network. In this perspective, the relative size of country $i$, as captured by $\theta_i$, is a measure of country $i$'s relative ‘attraction’ from the point of view of firm location. Analogously, although $\varphi_i$ is a bundle of the various trade freenesses $\phi_{ij}$, it can be naturally viewed as an inverse measure of country $i$’s relative ‘accessibility’ to all its trading partners (see expression (17) below).\(^3\)

For an interior equilibrium to arise, it must be that no single country can be at the same time too ‘attractive’ (large $\theta_i$) and too ‘accessible’ (small $\varphi_i$). In particular, a sufficient condition for a spatial equilibrium to be interior is that $\theta_i < \varphi_i$ holds for all countries $i = 1, 2, \ldots M$ (Behrens et al., 2005). Second, note that the equilibrium conditions (14) define a linear system with respect to the industry shares $\lambda^*$. This property is particularly

\(^3\)Note that $\varphi$ can be related to the Bonacich network centrality measure that has been recently used in non-cooperative network games (Ballester et al., 2006). To see this, note that $\Phi = \hat{\phi}I + G$, where $\hat{\phi} > 0$ is the common intracountry trade cost and where $G$ is the matrix of intercountry trade costs. Then

$$\Phi^{-1}1 = \varphi = \frac{1}{\hat{\phi}} \left[ I - \frac{1}{\hat{\phi}}(-G) \right]^{-1}1 = \frac{b(-G, \hat{\phi}^{-1})}{\hat{\phi}}$$

where $b(-G, \hat{\phi}^{-1})$ is the Bonacich measure associated with the network implied by $-G$ (the opposite of the trade cost matrix). Therefore, quite naturally, $\varphi$ is inversely linked to the Bonacich measure.
useful since it implies that there is a unique spatial equilibrium as in Krugman (1980).4

Behrens et al. (2005) have shown the following result:

**Proposition 1 (existence, uniqueness, stability)** When (11) holds, a unique and globally stable spatial equilibrium exists for all parameter values of the model. Furthermore, when $\theta_i < \varphi_i$ for $i = 1, 2, \ldots M$, this equilibrium is interior and given by

$$\lambda^* = \left[\text{diag}(\Phi^{-1}1)\Phi\right]^{-1} \theta,$$

or, component by component, by

$$\lambda^*_i = \sum_j \frac{f_{ji}}{\sum_k f_{kj}} \theta_j.$$

**Proof.** For the sake of clarity, we only derive the expression for the interior equilibrium in this paper. See Behrens et al. (2005, Appendix 2) for a proof of existence, uniqueness, and stability.

From (13), the conditions for an interior equilibrium are $\Phi\text{diag}(\Phi\lambda^*)^{-1}\theta = 1$. Some straightforward algebra then yields

$$\theta = \left[\Phi\text{diag}(\Phi\lambda^*)^{-1}\right]^{-1} 1$$

$$= \text{diag}(\Phi\lambda^*)\Phi^{-1} 1.$$ 

$$= \text{diag}(\Phi\lambda^*)\text{diag}(\Phi^{-1}1)1.$$ 

Since $\text{diag}(\Phi\lambda^*)$ and $\text{diag}(\Phi^{-1}1)$ are diagonal matrices, their product is commutative so that $\theta = \text{diag}(\Phi^{-1}1)\text{diag}(\Phi\lambda^*)1$. Hence

$$\text{diag}(\Phi^{-1}1)^{-1}\theta = \text{diag}(\Phi\lambda^*)1 = \Phi\lambda^*.$$ 

4This differs from the typical models of the ‘new economic geography’ in which multiple equilibria may arise due to the non-linearity of the underlying equilibrium conditions (see, e.g., Fujita et al., 1999). Such non-linearity stems from self-reinforcing agglomeration forces, such as expenditure mobility when firms move with workers (‘backward linkages’; Krugman, 1991) or reductions in the input price indices of intermediate goods when firms consume each others’ outputs (‘forward linkages’; Krugman and Venables, 1995). In our model, there are no ‘backward linkages’ because workers are internationally immobile and no ‘forward linkages’ because firms do not use intermediates. Hence, there is no room for self-reinforcing agglomeration forces, which explains why the equilibrium is unique for all parameter values.
which finally yields
\[ \lambda^* = \Phi^{-1} \text{diag}(\Phi^{-1} 1)^{-1} \theta = \left[ \text{diag}(\Phi^{-1} 1) \Phi \right]^{-1} \theta \]
and establishes expression (15). Expression (16) can finally be obtained using the symmetry of \( \Phi \).

Finally, we derive the indirect utility at the interior spatial equilibrium. The properties of the quantity and price indices (2) and (5) ensure that \( P_i D_i = \mu \) and \( H_i = 1 - \mu \), which allows us to rewrite the utility (1) in indirect form as follows:
\[ U_i = \mu^\mu (1 - \mu)^{1 - \mu} P_i^{-\mu} = \mu^\mu (1 - \mu)^{1 - \mu} P_i^{-\mu}. \]

Then, at the equilibrium prices (7), the price index becomes
\[ P_i^* = N \frac{\mu \sigma}{\sigma - 1} \left( \sum_j \lambda_j^* \phi_{ji} \right)^{\frac{1}{\sigma - 1}} \]
which yields
\[ U_i^* = \mu^\mu (1 - \mu)^{1 - \mu} N \frac{\mu \sigma}{\sigma - 1} \left( \sum_j \lambda_j^* \phi_{ji} \right)^{\frac{\mu}{\sigma - 1}}. \]

Using \( N = (\mu L)/(F \sigma) \), as well as expression (14) and the symmetry assumption \( \phi_{ij} = \phi_{ji} \), we finally obtain
\[ U_i = k \left( \sum_j \phi_{ji} \lambda_j^* \right)^{\frac{\mu}{\sigma - 1}} \frac{\theta_i}{\varphi_i^{\frac{\mu}{\sigma - 1}}} \],
where \( k > 0 \) is a constant bundle of parameters that does not depend on \( i \). Expression (17) shows that \( \theta_i/\varphi_i \) is a sufficient statistic to assess welfare changes: a large expenditure share \( \theta_i \) and a good access to world markets, i.e., a small value of \( \varphi_i \), both raise welfare in country \( i \). In other words, stronger attraction and better accessibility are both associated with higher welfare.

3 Spatial networks and trade frictions

Up to now, our model is very much in the spirit of standard CES trade models with an arbitrary number of countries. We now turn to the formal representation of the spatial
network linking the trading partners, as well as to a finer analysis of the trade frictions \( \tau_{ij} \) between them. A finer analysis of both the spatial network and the trade frictions is intended to capture additional real-world details because spatial interactions and trade impediments between countries are numerous and varied.

Following a standard approach in international trade and economic geography, we assume that countries are dimensionless points. When there are only two of them, which is the case usually considered in the literature, their relative position is irrelevant. Yet, when there are more than two countries, and when trade frictions are not pairwise symmetric across all countries, *their relative position becomes important* and must be somehow taken into account. We believe that a natural way of thinking about a multi-country world is in terms of spatial network, i.e., a graph.\(^5\) Each country can then be viewed as a node of the spatial network, which is linked to the other countries via edges (which represent transportation links). Formally, the space-economy is described by a graph \((\mathcal{M}, \mathcal{E})\), where \(\mathcal{M} = \{1, 2, \ldots, M\}\) is the set of nodes and \(\mathcal{E}\) the set of edges. In what follows, we denote by \((i, j) \in \mathcal{E}\) the edge linking nodes \(i\) and \(j\) in \(\mathcal{M}\) (which may not exist). Before proceeding, we need to define a few concepts that we will use in the subsequent analysis.

Two countries \(i\) and \(j\) for which there exists an edge will be called *neighbor countries*. A path between two nodes \(i\) and \(j\) is a sequence \(P = \{(i, k_1), (k_1, k_2), \ldots, (k_n, j)\}\) of edges linking them. A graph is said to be *connected* if there exists at least one path between any pair of nodes \((i, j) \in \mathcal{M} \times \mathcal{M}\). A graph is said to be *undirected* if every edge can be crossed equally in both directions. In what follows, we focus exclusively on connected and undirected graphs with at most one edge between each pair of nodes. As is well known, there are several special structures for these types of graphs (Harary, 1969). The following special structure will be particularly useful in the later developments.

**Definition 1 (local tree structure)** A graph is locally a tree around an edge \((i, j) \in \mathcal{E}\) if there exists a unique path connecting any pair of nodes in the subset of nodes \(\mathcal{M}'_{(i,j)} = \{l \in \mathcal{M}, \ (l, i) \in \mathcal{E} \text{ or } (l, j) \in \mathcal{E}\}\).

\(^5\)Note that the graph representation of a spatial structure has been extensively used in other fields like Operations Research and location theory.
In words, whenever a graph is locally a tree around an edge \((i, j)\), there are no cycles in the subset of neighboring nodes of \(i\) and \(j\).

In order to apply the graph-theoretic approach to a spatial economy, we need to make more precise how trade frictions affect flows between nodes. Denote by \(r_{ij}^{-1} \in (0, 1)\) the friction of edge \((i, j)\), which may be interpreted as the cost of crossing it: for one unit of any variety to reach country \(i\) from country \(j\), when using edge \((i, j)\), \(r_{ij}\) units have to be sent, the excess melting away en route. Hence, \(r_{ij}^{-1}\) may be viewed as a standard iceberg coefficient. Because the graph is undirected, \(r_{ij}^{-1} = r_{ji}^{-1}\) holds. We can think of \(r_{ij}\) as measuring all transport frictions between the two countries that arise due to, e.g., the existence of physical distance, or geographical features, or the transportation technology. Since countries are dimensionless points, intra-country transport frictions are assumed to be zero, i.e., \(r_{ii} = 1\) for all \(i \in \mathcal{M}\).

Shipping between two non-neighbor countries \(i\) and \(j\) occurs along a path \(\mathcal{P}\) of the spatial network linking the origin with the destination country. Arbitrage by profit-maximizing firms ensures that shipping occurs along the lowest cost route. Formally, let \(\mathcal{P}_{ij}\) denote the set of all paths between \(i\) and \(j\). Then, the transport friction between \(i\) and \(j\) is the friction calculated along the minimum cost path:\(^6\)

\[
\delta_{ij} \equiv \min_{\mathcal{P} \in \mathcal{P}_{ij}} \prod_{(l < m) \in \mathcal{P}} r_{lm}, \quad \text{with} \quad \prod_{(l < m) \in \mathcal{P}} r_{lm} \equiv r_{ik_1} r_{k_1 k_2} \cdots r_{k_n j}.
\]

Note that transport frictions are, in general, not origin-destination specific. In order to ship from country \(i\) to country \(j\), one has to use infrastructure in third countries along the path \(\mathcal{P}_{ij}\). Yet, the third-country frictions \(r_{kl}\) are not specific to the relationship between \(i\) and \(j\), since every other country shipping along this route to country \(j\) would have to incur it equally. In other words, transport frictions are, by definition, non-discriminatory in that no country can be excluded from changes in them.

\(^6\)It can be verified that the log of \(\delta\) constitutes an exponential network metric. Similar types of metrics are used in general relativity and quantum mechanics. This should not come as a surprise, since it is well known that the CES model gives rise to a ‘gravity equation’ (e.g., Anderson and van Wincoop, 2003). In our case, when \(r_{kl} \equiv e^{\xi_{kl}}\), the metric given by the log of \(\delta\) reduces to the standard network metric computed on the \(\xi_{kl}\) (see also Behrens et al., 2005, Appendix 2).
Although transportation related costs are still not negligible, transport frictions account only for one part of total trade frictions (Anderson and van Wincoop, 2004). The second component are what we call non-transport frictions. These frictions may include, without being exhaustive, tariff barriers, non-tariff barriers (red-tape, administrative delays, different product standards, sanitary and security requirements), and miscellaneous barriers (differences in languages, currency, and accounting standards). Note that, contrary to transport frictions, these non-transport frictions are mostly country-pair specific, i.e., discriminatory in nature. Indeed, raising tariffs in country $i$ for goods produced in country $j$ a priori does not apply to third countries $k$. Although the relative importance of transport and non-transport frictions is often unclear and depends on the trading partners, there is some evidence suggesting that non-transport frictions account for a larger part of total trade frictions.7

Let $t_{ij}$ stand for the ad valorem tariff equivalent of non-transport frictions between countries $i$ and $j$. Total trade frictions are then given by $\tau_{ij} = (1 + t_{ij})\delta_{ij}$, with $\tau_{ii} = 1$ since $t_{ii} = 0$ and $\delta_{ii} = 1$. Note that the transport friction $\delta_{ij}$ is independent of $t_{ij}$, i.e., changes in non-transport frictions do not induce changes in minimum cost paths. This property will be important for the rest of our analysis.8 Finally, we assume for simplicity that non-transport frictions are the same between two countries regardless of the directions of trade flows ($t_{ij} = t_{ji}$). When taken together with the undirected network assumption, this implies that $\tau_{ij} = \tau_{ji}$ so that the trade cost matrix is symmetric (as imposed in Section 2).

Before proceeding with our analysis, it is worth pointing out one important aspect

---

7In the European Union, for example, tariffs were already zero at the end of the 1960s, whereas direct frictional barriers remain relatively large even nowadays and are a matter of frequent dispute among member states (Baldwin and Wyplosz, 2004). Furthermore, the large residuals of the gravity equation (referred to as the ‘border effect’) suggest that crossing an international boundary significantly impedes trade flows.

8Note that this property only holds if, e.g., transhipment through a country does not require the payment of tariffs or any other impediments. Empirically, this seems to be the rule since tariffs are levied on goods destined to the local market only (either for intermediate or for final demand).
related to trade frictions. Indeed, the matrix of transport frictions, defined by the $\delta_{ij}$, and the matrix of non-transport frictions, defined by the $1 + t_{ij}$, differ in one fundamental respect: while the former satisfies by definition the triangle inequality, the latter usually need not do so. Consequently, the triangle inequality is not generally satisfied by the matrix of total trade frictions, defined by the $\tau_{ij}$ (resp., their ‘inverses’ $\phi_{ij}$). The reason is that non-transport frictions have no direct link with the topological properties of the spatial network, i.e., they work as if there were direct links between any pair of countries (all countries are neighbors). On the contrary, transport frictions have a direct link with the topological properties of the spatial network (via the network metric), and some countries are not neighbors. Most naturally, this fundamental difference implies that the effects of changes in trade frictions on industry distribution and welfare will be easier to assess in a world with mostly transport frictions, than in a world with mostly non-transport frictions.

4 Impacts of changes in trade frictions

We now analyze how changes in trade frictions affect countries’ equilibrium distribution of activity, as well as their welfare level. In other words, we are interested in assessing the impacts of changes to the freeness of trade matrix $\Phi$. In so doing we will, for simplicity and to make our point most clearly, focus on two ‘pure’ cases: (i) changes in non-transport frictions only; and (ii) changes in transport frictions only. Considering jointly changes in both types of frictions beclouds the analysis without adding much to our understanding of the underlying mechanisms.

4.1 The global impacts of changes in non-transport frictions

We first investigate how changes in non-transport frictions affect industry location and welfare, assuming pairwise symmetric transport frictions across all countries: $\delta_{ij} = \delta$ for

---

9See Behrens et al. (2005) for a detailed analysis on the impacts of changing market sizes $\theta$. 

---
all country pairs \((i, j)\).\(^{10}\) To highlight the additional insights of a multi-country setup when compared with a two-country world we deal, in particular, with preferential trade agreements. Put differently, we focus on bilateral changes in non-transport frictions and identify how they affect countries directly and indirectly.

In dealing with non-transport frictions, we assume that some ‘Rules of Origin’ apply: when delivered to country \(j\), a good is certified as ‘Made in country \(i\)’ and faces the friction \(t_{ij}\) only if a substantial part of its value added is generated in \(i\).\(^{11}\) This assumption rules out arbitrage on non-transport costs and formally implies that the triangle inequality holds \emph{with respect to the final destination market} (but not necessarily for the trade cost matrix as a whole). To see this, note that shipping from \(i\) to \(j\) costs \((1 + t_{ij})\delta_i\), whereas shipping via \(k\) costs \((1 + t_{ij})\delta_k > (1 + t_{ij})\delta_j\). In other words, changes in the non-transport friction \(t_{ij}\) do not spill over on the trade frictions faced by third countries.

The freeness of trade matrix \(\Phi\) is then given as follows:

\[
\Phi = \begin{pmatrix}
1 & \xi(1 + t_{12})^{1-\sigma} & \cdots & \xi(1 + t_{1M})^{1-\sigma} \\
\xi(1 + t_{21})^{1-\sigma} & 1 & \cdots & \xi(1 + t_{2M})^{1-\sigma} \\
\vdots & \vdots & \ddots & \vdots \\
\xi(1 + t_{M1})^{1-\sigma} & \xi(1 + t_{M2})^{1-\sigma} & \cdots & 1
\end{pmatrix},
\]

where \(\xi \equiv \delta^i - \sigma\) is a strictly positive constant that we can disregard in what follows.

In the case of \(M = 2\) countries, Krugman (1980) has shown that the larger country always attracts firms from the smaller country when the freeness of trade increases. Furthermore, such increase in the freeness of trade always raises welfare in both countries. In other words, preferential trade liberalization between two countries isolated from the rest of the world enhances the asymmetries between them but raises the welfare in each of them. The foregoing result does not hold in general when the two countries are part of a more general spatial network. This can be illustrated concisely by simple numerical examples. Assume that \(M = 3\) and that non-transport frictions \(t_{12}\) between countries 1

\(^{10}\)This assumption is not required for the results but makes their exposition much easier. The important assumption is that the \(\delta_{ij}\) are held fixed.

\(^{11}\)Note that ‘Rules of Origin’ usually apply in case of tariff barriers (e.g., Feenstra, 2004, ch.6).
and 2 decrease marginally, whereas those involving country 3 (i.e., \( t_{13} \) and \( t_{23} \)) remain unchanged. Clearly, countries 1 and 2 gain better reciprocal access to the their markets, whereas their freeness of trade with the rest of the world is unchanged.

Whether liberalizing countries gain in terms of firms and welfare depends on their initial attraction and accessibility. If countries 1 and 2 have the same attraction as the third country, but share superior accessibility, they both gain firms and welfare at the expense of 3. This is the standard result highlighted in the existing literature (see, e.g., Baldwin et al., 2003, equation (14.5)). For example, that would be the case if the international pattern of attraction and accessibility were described by

\[
\Phi = \begin{pmatrix} 1 & 0.1 & 0.1 \\ 0.1 & 1 & 0.5 \\ 0.1 & 0.5 & 1 \end{pmatrix}, \quad \varphi = \begin{pmatrix} 0.88 \\ 0.61 \\ 0.61 \end{pmatrix} \quad \text{and} \quad \theta = \begin{pmatrix} 0.33 \\ 0.33 \\ 0.33 \end{pmatrix} \tag{18}
\]

which implies that all countries have the same size and that the non-transport frictions between countries 2 and 3 are lower than those between them and country 1.

In general, however, both the number of firms and the level of welfare may fall in the integrating countries. First, firms may leave the integrating countries while their welfare still rises. To see this, suppose that expenditure in the high non-transport friction country 1 is larger, whereas trade frictions remain unchanged:

\[
\Phi = \begin{pmatrix} 1 & 0.1 & 0.1 \\ 0.1 & 1 & 0.5 \\ 0.1 & 0.5 & 1 \end{pmatrix}, \quad \varphi = \begin{pmatrix} 0.88 \\ 0.61 \\ 0.61 \end{pmatrix} \quad \text{and} \quad \theta = \begin{pmatrix} 0.8 \\ 0.1 \\ 0.1 \end{pmatrix}. \tag{19}
\]

In this case preferential trade liberalization between countries 1 and 2 would decrease the number of firms in the integrating area while increasing it in the rest of the world. This may be explained as follows. Consider definition (10), with \( w_i = w_j = w_k = 1 \), which predicts firms’ profits in country \( i \) for any given distribution \( \lambda \) of firms. A marginal increase in \( \phi_{12} \), taking \( \lambda \) as given, raises both the numerator and the denominator of RMP\(_i\) for \( i = 1, 2 \). The former effect stems from the fact that firms in the integrating countries enjoy freer reciprocal access to each other’s consumers. The latter effect stems from the fact that firms in the integrating countries also suffer from tougher competition.
from each other’s firms. In this example, the former effect is dominated by the latter, so that some firms must leave the integrating area in equilibrium.\footnote{It is worth noting that this result is highly reminiscent of the ‘merger paradox’. When two countries ‘merge’, their ‘market share’ (i.e., their mass of firms), may decrease.}

Second, welfare may even fall in the integrating countries. Suppose that the initial trade freeness matrix and the expenditure share vector in (18) are changed so that

\[
\Phi = \begin{pmatrix} 1 & 0.4 & 0.1 \\ 0.4 & 1 & 0.5 \\ 0.1 & 0.5 & 1 \end{pmatrix}, \quad \varphi = \begin{pmatrix} 0.81 \\ 0.29 \\ 0.77 \end{pmatrix} \quad \text{and} \quad \theta = \begin{pmatrix} 0.4 \\ 0.2 \\ 0.4 \end{pmatrix}.
\]

Then, we obtain that welfare falls in country 1 and rises in country 2, implying that one of the integrating countries experiences welfare losses, which never occurs in the case of \( M = 2 \). This result is driven by the presence of the hub effect: being geographically advantageously located, country 2 attracts firms and gains welfare at the expense of the peripheral country 1. It is reminiscent of the ‘two-tier home market effect’ of Baldwin et al. (2003): firms are attracted towards the integrating block (‘first tier’) but they pick the larger country of the block (‘second tier’). The difference is that in our case changes in the integrating block are driven by accessibility differences instead of unequal market size (\( \theta \) is held fixed here).

Although general theoretical results are hard to come by, we can derive one for the special case of \( M = 3 \) countries. Indeed, the next proposition shows that when \( M = 3 \) the excluded country always loses: \( \partial U^*_3 / \partial \phi_{12} < 0 \) at any interior equilibrium.\footnote{Note, that Proposition 2 holds for interior solutions only. Once we take into account the fact that trade integration may well lead to corner solution, the analysis become even more involved. We present an example of such a case in Appendix 1.}

**Proposition 2 (harmful exclusion)** When there are \( M = 3 \) countries, reducing non-transport frictions between two countries always harms the excluded one, provided the equilibrium distributions before and after integration are interior.
Proof. It is readily verified that the numerator of $\partial U_3^*/\partial \phi_{12}$ is quadratic in $\phi_{12}$, while the denominator is positive. Solving $\partial U_3^*/\partial \phi_{12} = 0$ yields

$$\phi_{12} = \frac{\phi_{13} - \phi_{13}^2 + \phi_{23} - \phi_{23}^2 \pm (\phi_{13} - \phi_{23}) \sqrt{-2 (1 - \phi_{13}) (1 - \phi_{23}) (\phi_{13} + \phi_{23})}}{\phi_{13} + \phi_{23} - 2 \phi_{13} \phi_{23}},$$

which is clearly a complex number since $\phi_{13}, \phi_{23} \in (0, 1)$. Hence, $\partial U_3^*/\partial \phi_{12}$ does not change sign. One can verify that $\text{sgn}(\partial U_3^*/\partial \phi_{12}) = -\text{sgn}(\partial \varphi_3/\partial \phi_{12}) < 0$ when evaluated at $\phi_{12} = \phi_{13} = \phi_{23} \in (0, 1)$. Hence, the derivative is always negative, which proves the result. ■

To sum up, there are complex mechanisms at work when there are more than two countries. This complexity should not come as a surprise since it is well known in transportation science that changes in network structure or edge capacities have multiple implications and may yield counterintuitive results (see, e.g., the so-called Braess paradox; Braess, 1968). The three examples presented in the foregoing, as well as that developed in Appendix 1, suffice to show that even in the simplest setting with $M = 3$, general results are extremely difficult to establish from a theoretical point of view without imposing at least some restrictions on $\Phi$ and $\theta$. Whether ‘realistic’ examples, using plausible values for $\theta$, $\Phi$ and $\sigma$, also may give rise to such counterintuitive results is not a priori clear. To answer this question would require extensive numerical analysis and simulation of counterfactuals using a real-world data calibrated version of the model, a task that we reserve for future work.

4.2 The local impacts of changes in transport frictions

As shown in the previous section, it is hardly possible to obtain general results on the impact of changes in non-transport frictions when there are more than two countries. The underlying reason is twofold: (i) the network may be viewed as fully connected, i.e., the matrix $\Phi$ has $M \times M$ independent parameters; and (ii) there is, in general, no relationship between $\Phi$ and the topology of the network since $\Phi$ does not derive from a metric. Yet, if we abstract from non-transport frictions and solely focus on transport frictions, the natural restrictions on $\Phi$ deriving from the metric can be used to obtain a richer set of
results. Furthermore, the matrix $\Phi$ has less than $M \times M$ independent parameters, which also significantly simplifies the analysis.\(^{14}\)

To illustrate how changes in transport frictions affect the distribution of industry and the level of welfare, we sterilize differences in non-transport frictions by assuming $t_{ij} = \bar{t}$. Hence, changes in trade frictions $\tau_{ij}$ are due to changes in transport frictions $\delta_{ij}$ only.\(^{15}\) This implies that, because $\delta$ is a metric, the triangle inequality holds both with respect to the destination market and for the trade cost matrix $(\tau_{ij})$: formally, $(1 + \bar{t})\delta_{ij} \leq (1 + \bar{t})^2 \delta_{ik} \delta_{kj}$. The trade freeness matrix $\Phi$ is then given by:

$$
\Phi = \begin{pmatrix}
1 & \xi' \delta_{12}^{1-\sigma} & \ldots & \xi' \delta_{1M}^{1-\sigma} \\
\xi' \delta_{21}^{1-\sigma} & 1 & \ldots & \xi' \delta_{2M}^{1-\sigma} \\
\vdots & \vdots & \ddots & \vdots \\
\xi' \delta_{M1}^{1-\sigma} & \xi' \delta_{M2}^{1-\sigma} & \ldots & 1
\end{pmatrix},
$$

where $\xi' \equiv (1 + \bar{t})^{1-\sigma}$ is a strictly positive constant that we can disregard in what follows. Note that the graph associated with this matrix need not be fully connected, because there may be no edges between some country pairs (think, e.g., of shipping from Germany to Spain, which requires passing through France). The reason is that the $\delta_{ij}$ are minimum cost paths, which in general do not have direct links between the origin and the destination countries (except when they are neighbors). As a result, the impact of changing $r_{kl}$ (as given in Section 3) on the spatial equilibrium will strongly depend on the topological properties of the transportation network in the vicinity of the edge subject to change.

One of the strongest structures a graph can exhibit is that of a tree, which is an undirected graph in which there is a unique path between any pair of nodes (Harary, 1969). Although the strict assumption of a tree is too strong with respect to real-world spatial networks, many networks have a structure that closely mimics that of a tree.\(^{16}\)

\(^{14}\)In the case of a tree there are, for example, $3M - 2 < M^2$ independent parameters when $M > 2$. The reduction in the number of independent parameters becomes very significant for large networks.

\(^{15}\)As in the previous section, symmetry is not generally required for the results to hold. It only makes their exposition much easier. The key assumption is that the $t_{ij}$’s are held fixed.

\(^{16}\)In particular, all multi-hub networks are tree-like structures, e.g., the international airline network for both passengers and cargo. It is also of interest to note that tree-like networks emerge as the optimal
Figure 1 depicts an example of a graph with 9 nodes (countries) and 10 edges (shipping routes).

Insert Figure 1 about here.

As can be seen from Figure 1, the ‘left part’ of the graph (nodes 1, 2, 3, 4 and 5) form a tree-like structure, whereas the ‘right part’ displays a more complex pattern. Recalling Definition 1 in Section 3, one can verify that the graph depicted in Figure 1 is locally a tree around the edges (1,3), (2,3), (3,4), and (4,5), whereas it is not around any other edge. Note also that the local tree property implies that the edge \((i, j)\) is a bottleneck of the graph, since some trade flows must necessarily flow through this edge no matter the (finite) value of transport friction \(r_{ij}\).

By Definition 1, a sub-graph forming locally a tree differs from a general spatial network in that there are no cycles, i.e., shipping routes are always uniquely determined. For instance, shipping from country 1 to country 6 necessarily implies transhipment through the transit countries 3 and 4, which then implies that \(\phi_{16} = \phi_{13}\phi_{34}\phi_{46}\). This corresponds to an extreme case where the triangle inequality \(\tau_{ij} \leq \tau_{ik}\tau_{kj}\), or equivalently \(\phi_{ij} \geq \phi_{ik}\phi_{kj}\), holds as an equality. Moreover, there is no room for international arbitrage since no firm in country \(k\) can earn profits by importing goods from country \(i\) and re-exporting them to country \(j\).

Although, as shown in Section 4.1, no general results on the effects of economic integration can be derived in a general spatial network with respect to changes in non-transport frictions, we can prove three strong results relating to changes in the transport frictions when the network is locally a tree at \((i, j)\). To begin with, we prove that the impact of a change in \(\phi_{ij}\) is localized.

**Lemma 1 (localized impacts of changes in transport frictions)** Assume that the spatial network \((\mathcal{M}, \mathcal{E})\) is locally a tree at \((i, j)\), and let \(M'_{(i,j)}\) denote the subset of neighbors. This is in particular so when crossing nodes involves additional costs as in the cases of re-containerization in ports or connecting flights in airports.
boring nodes of $i$ and $j$. Then
\[
\frac{\partial \lambda^*_i}{\partial r_{ij}} > 0
\]
for all $l \in M_{(i,j)} \setminus \{i, j\}$, whereas
\[
\frac{\partial \lambda^*_k}{\partial r_{ij}} = 0
\]
for all $k \in M \setminus M_{(i,j)}$.

Proof. See Appendix 2. ■

Lemma 1 shows that when the spatial structure is locally described by a tree, reductions in transport frictions matter most for nearby countries. In particular, such improvements always reduce the firm shares in excluded neighboring countries without affecting excluded non-neighboring countries. In other words, whereas changes in non-transport frictions have global effects, changes in transport frictions have only local effects in a tree. For example, in Figure 1, when $(i, j) = (3, 4)$ we have $M'(3, 4) = \{1, 2, 3, 4, 5, 6\}$. A change in $\phi_{45}$ ‘spills over’ on $\lambda^*_4$ to $\lambda^*_6$, while the remaining shares $\lambda^*_7$ to $\lambda^*_{11}$ are left unchanged.

Lemma 1 allows us to prove the following result:

Proposition 3 (inflow of firms) Assume that the spatial network $(\mathcal{M}, \mathcal{E})$ is locally a tree at $(i, j)$. Then
\[
\frac{\partial (\lambda^*_i + \lambda^*_j)}{\partial r_{ij}} < 0.
\]

Proof. When transport frictions are reduced between $i$ and $j$, by Lemma 1 all neighboring countries lose some industry, whereas the other countries are unaffected. Since the shares $\lambda^*_i$ sum to one, it then must be that the integrating block increases its industry share at the expense of the excluded countries. ■

Proposition 3 states that reductions in transport frictions increase the total industry share $\lambda^*_i + \lambda^*_j$ of the integrating countries if the spatial network is locally a tree. Note, however, that the individual shares $\lambda^*_i$ and $\lambda^*_j$ do not necessarily both rise. If $\lambda^*_i$ is much larger than $\lambda^*_j$, the former rises and the latter falls. The reason is the two-tier home-market effect, which is a typical manifestation of the result that the shorter the distance
from a large country, the greater the decrease in the numbers of firms in neighboring countries. Accordingly, reductions in transport frictions between large and small countries can decrease the attractiveness of the small country. We may call such a phenomenon a ‘straw effect’, because economic activities migrate to large countries through new highways and infrastructure as juice in a glass is sucked up by a straw.

Our third and last result concerns the welfare effects of reductions in transport frictions when the network is locally a tree:

**Proposition 4 (Pareto improvement)** Assume that the spatial network \((\mathcal{M}, \mathcal{E})\) is locally a tree at \((i, j)\). A decrease in the transport friction \(r_{ij}\) raises the welfare of both countries \(i\) and \(j\), whereas it does not change the welfare of all the remaining countries.

**Proof.** See Appendix 3.

Proposition 4 shows that, despite the straw effect, consumers in both integrating countries \(i\) and \(j\) are always better off due to reductions in transport frictions \(r_{ij}\). That is, the increase in the freeness of trade is not only jointly desirable for the two countries, but also Pareto dominant in spite of the fact that one of the countries may lose firms. The fact that welfare in third countries is unaffected contrasts with the result obtained when trade integration happens through the reduction of non-transport frictions (Proposition 2). This difference is due to the fact that reductions in transport frictions between any two countries \(i\) and \(j\) cannot be ‘preferential’, as improved links are used also for shipments to and from all other countries.

To understand the intuition underlying this result, consider Figure 1. For simplicity, assume that \(\phi_{ij} = 1/5\) for all edges \((i, j)\) and that \(\theta_i = 1/9\) for all nodes. The spatial equilibrium and the associated utilities are then given by

\[
\lambda^* = (0.09 \ 0.09 \ 0.15 \ 0.12 \ 0.09 \ 0.17 \ 0.08 \ 0.08 \ 0.11)
\]

\[
u^* = (0.13 \ 0.13 \ 0.22 \ 0.22 \ 0.13 \ 0.26 \ 0.15 \ 0.15 \ 0.19).
\]

Countries 3, 4 and 6 naturally attract more industry due to their ‘central’ location in the network. Note also that country 4 has less industry, since it lies in the ‘shadow’ of
countries 3 and 6. Yet, having good consumer access to the varieties produced in both countries 3 and 6, it offers the same utility as country 3 in equilibrium. Consider now an improvement in the link between countries 3 and 4 and assume that $\phi_{34}$ increases to 2/5. This will drain firms from the neighboring countries 1, 2, 5 and 6, whereas the non-neighboring countries 7, 8 and 9 are unaffected. Straightforward computation shows that the new spatial equilibrium and the new utilities are given by

$$\lambda^* = (0.08 \ 0.08 \ 0.18 \ 0.15 \ 0.08 \ 0.16 \ 0.08 \ 0.08 \ 0.11)$$

$$u^* = (0.13 \ 0.13 \ 0.29 \ 0.29 \ 0.13 \ 0.26 \ 0.15 \ 0.15 \ 0.19).$$

How can we explain that a country loses industry yet sees its welfare unaffected? Consider the case of country 5. There is a negative and a positive effect of an increase in $\phi_{34}$. First, there is the loss of local industry, which decreases consumer welfare since these varieties must now be imported. This loss of industry is triggered by the relocation of competing firms from countries 1, 2 and 6 towards the better accessible countries 3 and 4, which makes competition for firms in country 5 in these markets fiercer, and thus less profitable. Second, there is the positive effect of an overall better access to all varieties produced in countries 1, 2, 3 and 4, due to the relocation of firms. This makes imports cheaper and, therefore, raises welfare in country 5. In equilibrium, these two effects offset each other, so that welfare in country 5 does not change. Stated differently, average consumer access to all varieties remains the same, despite the relocation of domestic firms. The crucial difference with changes in non-transport barriers, as analyzed in Section 4.1, is that country 5 also benefits from the change in $\phi_{34}$ because this improvement is not specific to countries 3 and 4. Could country 5 be excluded from this improvement, only the negative effect would survive and the change would not be Pareto improving.

Note, finally, that this strong result does not hold for link improvements that occur on portions of the graph where it is not locally a tree, such as $\phi_{46}$ and $\phi_{67}$. This is because the presence of cycles generates feedbacks in the network which do affect third countries. These results have been highlighted in Section 4.1 using some simple examples with $M = 3$.  

25
5 Conclusion

We have analyzed how decreases in transport and non-transport costs affect the location of firms, the structure of trade, and the welfare of nations. Our multi-country model with transportation network reveals that the consequences of deepening economic integration are very likely to depend on which components of trade costs actually decrease. Whereas changes in non-transport costs seem to have global impacts, the impacts of reductions in transport costs are likely to be localized.

These findings have important policy implications. In a comparable model with freely mobile capital, Baldwin et al. (2003, Ch.14.3) show that a custom union unambiguously leads to an inflow of industry into the union, welfare gains for the members, and welfare losses for the non-members. We have shown that those clear-cut results stem from: (i) a common external tariff that custom unions impose whereas preferential trade agreements do not; (ii) identical transport costs between any pair of countries. Otherwise, preferential trade liberalization may harm not only excluded but also some member countries. This suggests that successful preferential trade agreements also require at least some coordination with respect to infrastructural projects.

For example, due to differential transport costs, European integration may have a negative impact not only on excluded countries but also on small or remote member countries. Accordingly, contrary to standard beliefs, eastern enlargement might be detrimental not to incumbent members but rather to the newcomers, which are all characterized by far smaller markets. Even if factor price differences may lead to the relocation of standardized production processes to the new member countries, market-size dependent R&D, services or knowledge intensive production processes may well relocate to the incumbent countries, thereby harming long-run development of the newcomers. Analyzing these important questions requires dropping the assumption of factor price equalization, which is left for future work.

Acknowledgements. We wish to thank two anonymous referees, Takashi Akamatsu, Johannes Bröcker, Masa Fujita, Nils Happich, Philippe Monfort, Pierre M. Picard, Ya-
suhiro Sato, Jens Südekum, Yves Zenou and participants at the “Summer Workshop for Trade and Location” in Kiel (Germany), and the “International Symposium on Spatial Economics and Transportation” in Sendai (Japan) for valuable comments and suggestions. Kristian Behrens gratefully acknowledges financial support from the European Commission under the Marie Curie Fellowship MEIF-CT-2005-024266. Gianmarco Ottaviano gratefully acknowledges financial support from MIUR. Takatoshi Tabuchi gratefully acknowledges financial support from the Japanese Ministry of Education and Science (Grant-in-Aid for Science Research 13851002 and 18203012). The views expressed in this paper are those of the authors and do not necessarily reflect those of the Bank of Italy. The usual disclaimer applies.

References


[16] Harary, F., 1969, Graph Theory (Reading: Addison-Wesley).


**Appendix 1: Changes in non-transport costs and corner solutions**

Consider the following simple example with $M = 3$ countries. Before integration, the economy is described by

\[
\Phi^{(1)} = \begin{pmatrix}
1 & 0.795 & 0.7 \\
0.795 & 1 & 0.5 \\
0.7 & 0.5 & 1
\end{pmatrix}
\quad \text{and} \quad
\theta^{(1)} = \begin{pmatrix}
0.01 \\
0.40 \\
0.59
\end{pmatrix}.
\]

Assume that countries 1 and 2 remove all barriers to trade. Hence, everything works as if they would form only a single country after integration.\(^{17}\) Their access to country 3 is

\(^{17}\)Note that the new freeness of trade matrix can neither be described by

\[
\Phi^{(2)} = \begin{pmatrix}
1 & 1 & 0.7 \\
1 & 1 & 0.7 \\
0.7 & 0.7 & 1
\end{pmatrix}
\quad \text{nor by} \quad
\Phi^{(2)} = \begin{pmatrix}
1 & 1 & 0.7 \\
1 & 1 & 0.5 \\
0.7 & 0.5 & 1
\end{pmatrix}.
\]
given by \( \max\{\phi_{13}, \phi_{23}\} = \phi_{13} = 0.7 \), so that after integration

\[
\Phi^{(2)} = \begin{pmatrix}
1 & 0.7 \\
0.7 & 1
\end{pmatrix}
\quad \text{and} \quad
\theta^{(2)} = \begin{pmatrix}
0.41 \\
0.59
\end{pmatrix}.
\]

It is readily verified that

\[
\left(\lambda_1^{(1)}, \lambda_2^{(1)}, \lambda_3^{(1)}\right) = (0.07, 0.17, 0.76) \quad \text{and} \quad \left(\lambda_1^{(2)}, \lambda_3^{(2)}\right) = (0, 1).
\]

Hence, the equilibrium is interior before integration, but corner after. In particular, \textit{the integrating area loses its industry}. To see that Proposition 2 does not apply in this case, we can compute the sufficient statistic for utility \( u_i^* = \sum_j \phi_{ij} \lambda_j^* \) as given by (17). It is readily verified that

\[
\left(u_1^{(1)}, u_2^{(1)}, u_3^{(1)}\right) = (0.74, 0.61, 0.89) \quad \text{and} \quad \left(u_{1+2}^{(2)}, u_3^{(2)}\right) = (0.70, 1.00).
\]

This shows that countries 1 and 2 lose firms, whereas country 1 loses and country 2 gains welfare. Furthermore, country 3 clearly gains both firms and welfare, which shows that Proposition 2 does not apply to corner solutions.

\textbf{Appendix 2: Proof of Lemma 1}

Assume that the transportation network is locally a tree at \((i, j)\). We compute in the following order: (i) \( f_{ik} \) with \((i, k) \notin \mathcal{E} \); (ii) \( f_{in} \) with \((i, i_n) \in \mathcal{E} \); (iii) \( f_{ii} \), in order to (iv) obtain \( \partial \lambda_k^* / \partial r_{ij} \); and (v) \( \partial \lambda_{in}^* / \partial r_{ij} \). Let \( i_1, \ldots, i_m \) be the set of countries that have an indirect link with \( j \) via \( i \), and let \( j_0, \ldots, j_m \) be the set of countries that have an indirect link with \( i \) via \( j \). Without loss of generality, we may set \( k = j_0 \), assume that \((j, j_0) \in \mathcal{E} \), and order countries as follows: \( i, j, i_1, \ldots, i_m, k(= j_0), j_1, \ldots, j_m' \). Hence, the

In both cases, \(|\text{diag}((\Phi^{(2)})^{-1}1)| = 0\) so that we cannot compute the equilibrium. This is because the new equilibrium will be a corner solution, whereas the matrix equations in Proposition 1 characterize interior equilibria only.
The trade freeness matrix is given by

\[
\Phi = \begin{pmatrix}
1 & \phi_{ij} & \phi_{i1} & \cdots & \phi_{im} & \phi_{ik} & \phi_{ij1} & \cdots & \phi_{ijm'} \\
\phi_{ji} & 1 & \phi_{j1} & \cdots & \phi_{jm} & \phi_{jk} & \phi_{ji1} & \cdots & \phi_{jjm'} \\
\phi_{i1i} & \phi_{i1j} & 1 & \cdots & \phi_{i1im} & \phi_{i1k} & \phi_{i1j1} & \cdots & \phi_{i1jm'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots \\
\phi_{im'i} & \phi_{im'j} & \phi_{im'i1} & \cdots & 1 & \phi_{im'k} & \phi_{im'j1} & \cdots & \phi_{im'jm'} \\
\phi_{ki} & \phi_{kj} & \phi_{ki1} & \cdots & \phi_{kim} & \phi_{kj1} & \phi_{km'} & \cdots & \phi_{km'} \\
\phi_{ji1} & \phi_{ji1j} & \phi_{ji1i1} & \cdots & \phi_{ji1im} & \phi_{ji1k} & 1 & \cdots & \phi_{ji1jm'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \cdots & \vdots \\
\phi_{j1m'} & \phi_{j1m'j} & \phi_{j1m'i1} & \cdots & \phi_{j1m'k} & \phi_{j1m'j1} & \cdots & 1 \\
\end{pmatrix}
\]

(i) The cofactor \( f_{ik} \) is the signed determinant of the submatrix extracted from \( \Phi \) by deleting row \( i \) and column \( k \). Dropping the multiplicative coefficient \((-1)^{i+m+4}\), it is given by

\[
\begin{vmatrix}
\phi_{ji} & 1 & \phi_{ji1} & \cdots & \phi_{ji1m} & \phi_{ji1j} & \cdots & \phi_{ji1jm'} \\
\phi_{i1i} & \phi_{i1j} & 1 & \cdots & \phi_{i1im} & \phi_{i1i1} & \cdots & \phi_{i1i1m'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\
\phi_{im'i} & \phi_{im'j} & \phi_{im'i1} & \cdots & 1 & \phi_{im'j1} & \cdots & \phi_{im'jm'} \\
\phi_{ki} & \phi_{kj} & \phi_{ki1} & \cdots & \phi_{kim} & \phi_{kj1} & \cdots & \phi_{km'} \\
\phi_{ji1} & \phi_{ji1j} & \phi_{ji1i1} & \cdots & \phi_{ji1im} & \phi_{ji1k} & 1 & \cdots & \phi_{ji1jm'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\
\phi_{j1m'} & \phi_{j1m'j} & \phi_{j1m'i1} & \cdots & \phi_{j1m'k} & \phi_{j1m'j1} & \cdots & 1 \\
\end{vmatrix}
\]

Since \( r_{ki} = r_{kj}r_{ji} \) when the network is locally a tree around \((i,j)\), we have that \( \phi_{ki} = \phi_{kj}\phi_{ji} \). Therefore, we can multiply row \( j \) by \( \phi_{kj} \) and subtract it from row \( k \). This operation yields 0 for the first \( i_m + 2 \) elements. Applying the same transformation to rows \( k, j_1, \ldots, j_m' \) gives the same result. Likewise, we can multiply column \( j \) by \( \phi_{ji} \) and subtract it from column \( j_1 \), which yields 0 for the first \( i_m + 1 \) elements. Applying the same transformation to columns \( j_2, \ldots, j_m' \) gives the same result. Putting together all these operations shows that, dropping the multiplicative constant \((-1)^{i+m+4}\), \( f_{ik} \) can be
rewritten as follows:

\[
\begin{pmatrix}
\phi_{ji} & 1 & \phi_{ji1} & \cdots & \phi_{ji(m-1)} & 0 & \cdots & 0 \\
\phi_{i1i} & \phi_{i1j} & 1 & \cdots & \phi_{i1(i+m-1)} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\phi_{i(m-1)i} & \phi_{i(m-1)j} & \phi_{i(m-1)i1} & \cdots & \phi_{i(m-1)(i+m-1)} & 0 & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 0 & * & \cdots & * \\
0 & \cdots & \cdots & \cdots & 0 & * & \cdots & * \\
0 & \cdots & \cdots & \cdots & 0 & * & \cdots & * \\
0 & \cdots & \cdots & \cdots & 0 & \cdots & \cdots & * \\
0 & \cdots & \cdots & \cdots & 0 & \cdots & \cdots & * \\
\end{pmatrix},
\]

where * denotes a non-zero element whose exact expression is of no particular interest to us. Rewrite \( \Phi \) in block form as follows:

\[
\Phi = \begin{pmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{21} & \Phi_{22}
\end{pmatrix},
\]

where

\[
\Phi_{11} \equiv \begin{pmatrix}
\phi_{ji} & 1 & \phi_{ji1} & \cdots & \phi_{ji(m-1)} \\
\phi_{i1i} & \phi_{i1j} & 1 & \cdots & \phi_{i1(i+m-1)} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\phi_{i(m-1)i} & \phi_{i(m-1)j} & \phi_{i(m-1)i1} & \cdots & \phi_{i(m-1)(i+m-1)}
\end{pmatrix},
\]

\[
\Phi_{12} \equiv \begin{pmatrix}
\phi_{ji(m-1)} & 0 & \cdots & 0 \\
\phi_{i1(i+m-1)} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
1 & 0 & \cdots & 0
\end{pmatrix},
\]

\[
\Phi_{22} \equiv \begin{pmatrix}
0 & * & \cdots & * \\
0 & * & \cdots & * \\
0 & * & \cdots & * \\
0 & * & \cdots & * \\
\end{pmatrix}.
\]

and where \( \Phi_{21} \) is \((1 + m') \times (1 + m)\) zero matrix. Using the Schur complement (see Horn and Johnson, 1985, p.22), we have

\[
f_{ik} = |\Phi_{11}| \left| \Phi_{22} - \Phi_{21} (\Phi_{11})^{-1} \Phi_{12} \right| = |\Phi_{11}| \left| \Phi_{22} \right| = 0,
\]

because \( |\Phi_{21}| = |\Phi_{22}| = 0 \).
The determinant $|\Phi|$ can be rewritten as
\[
\begin{vmatrix}
\phi_{ji} & 1 & \phi_{ji2} & \cdots & \phi_{jim} & \phi_{jk} & \phi_{jj1} & \cdots & \phi_{jjm'} \\
1 & \phi_{ij} & \phi_{i1i} & \cdots & \phi_{im} & \phi_{ik} & \phi_{ij1} & \cdots & \phi_{ijm'} \\
\phi_{i1i} & \phi_{i1j} & 1 & \cdots & \phi_{i1im} & \phi_{i1k} & \phi_{i1j1} & \cdots & \phi_{i1jm'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\phi_{im'i} & \phi_{im'j} & \phi_{im'i1} & \cdots & \phi_{im'im} & \phi_{im'k} & \phi_{im'j1} & \cdots & 1 \\
\end{vmatrix}
\]
where the first expression is obtained by exchanging the first two columns, and the second expression is obtained by dividing column $j$ by $\phi_{ij}$ and subtracting it from column $i$. 

\[
\begin{vmatrix}
\phi_{ji} - 1/\phi_{ij} & 1 & \phi_{ji2} & \cdots & \phi_{jim} & \phi_{jk} & \phi_{jj1} & \cdots & \phi_{jjm'} \\
0 & \phi_{ij} & \phi_{i1i} & \cdots & \phi_{i1m} & \phi_{i1k} & \phi_{i1j1} & \cdots & \phi_{i1jm'} \\
0 & \phi_{i1j} & 1 & \cdots & \phi_{i1im} & \phi_{i1k} & \phi_{i1j1} & \cdots & \phi_{i1jm'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \phi_{im'j} & \phi_{im'i1} & \cdots & 1 & \phi_{im'm} & \phi_{im'j1} & \cdots & \phi_{im'jm'} \\
\phi_{kj} (\phi_{ij} - 1/\phi_{ij}) & \phi_{kj} & \phi_{k1i} & \cdots & \phi_{k1m} & 1 & \phi_{kj1} & \cdots & \phi_{kjm'} \\
\phi_{j1j} (\phi_{ij} - 1/\phi_{ij}) & \phi_{j1j} & \phi_{j1i1} & \cdots & \phi_{j1im} & \phi_{j1k} & 1 & \cdots & \phi_{j1jm'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\phi_{jm'j} (\phi_{ij} - 1/\phi_{ij}) & \phi_{jm'j} & \phi_{jm'i1} & \cdots & \phi_{jm'm} & \phi_{jm'k} & \phi_{jm'j1} & \cdots & 1 \\
\end{vmatrix}
\]

... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ... ...

\[
\begin{vmatrix}
\phi_{ii} & \phi_{i1i} & \cdots & \phi_{i1m} & \phi_{i1k} & \phi_{i1j1} & \cdots & \phi_{i1jm'} \\
\phi_{i1i} & \phi_{i1j} & 1 & \cdots & \phi_{i1im} & \phi_{i1k} & \phi_{i1j1} & \cdots & \phi_{i1jm'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
\phi_{im'i} & \phi_{im'j} & \phi_{im'i1} & \cdots & \phi_{im'im} & \phi_{im'k} & \phi_{im'j1} & \cdots & 1 \\
\phi_{im'j} & \phi_{im'j} & \phi_{im'i1} & \cdots & \phi_{im'im} & \phi_{im'k} & \phi_{im'j1} & \cdots & 1 \\
\end{vmatrix}
\]
Factorizing the constant term in the first column, this is equal to

\[
\frac{1 - \phi_{ij}^2}{\phi_{ij}} \begin{vmatrix}
1 & 1 & \phi_{ji1} & \cdots & \phi_{ji m} & \phi_{jk} & \phi_{jj1} & \cdots & \phi_{jj m'} \\
0 & \phi_{ij} & \phi_{ii1} & \cdots & \phi_{ii m} & \phi_{ik} & \phi_{ij1} & \cdots & \phi_{ij m'} \\
0 & \phi_{11j} & 1 & \cdots & \phi_{11 i m} & \phi_{11 k} & \phi_{11 j1} & \cdots & \phi_{11 j m'} \\
n & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \phi_{i m j} & \phi_{i m i1} & \cdots & 1 & \phi_{i m k} & \phi_{i m j1} & \cdots & \phi_{i m j m'} \\
& \phi_{kj} & \phi_{kj} & \phi_{ki1} & \cdots & \phi_{ki m} & 1 & \phi_{kj1} & \cdots & \phi_{kj m'} \\
& \phi_{j1j} & \phi_{j1j} & \phi_{j1 i1} & \cdots & \phi_{j1 i m} & \phi_{j1 k} & 1 & \cdots & \phi_{j1 j m'} \\
& \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
& \phi_{j m' j} & \phi_{j m' j} & \phi_{j m' i1} & \cdots & \phi_{j m' i m} & \phi_{j m' k} & \phi_{j m' j1} & \cdots & 1 \\
\end{vmatrix}
\]

\[
= \frac{1 - \phi_{ij}^2}{\phi_{ij}} \left( -f_{ij} - \phi_{kj} f_{ik} - \sum_{n=1}^{m'} \phi_{k j n} f_{ij n} \right).
\]

However, we have shown in (i) above that the cofactors of the last two terms in the parentheses are zero. Thus, we can express the cofactor as follows:

\[
f_{ij} = -\frac{\phi_{ij}}{1 - \phi_{ij}^2} |\Phi|.
\]

Since this equality also holds for all \((i, i_n) \in \mathcal{E} (n = 1, 2, \ldots, m)\), we have

\[
f_{ii n} = -\frac{\phi_{ij}}{1 - \phi_{ij}^2} |\Phi|.
\]
(iii) Multiplying column \(j\) by \(\phi_{ij}\) and subtracting it from column \(i\), we have

\[
|\Phi| = \begin{vmatrix}
1 - \phi_{ij}^2 & \phi_{ij} & \phi_{i1} & \ldots & \phi_{im} & \phi_{ij1} & \ldots & \phi_{ijm'} \\
0 & 1 & \phi_{j1} & \ldots & \phi_{jm} & \phi_{j11} & \ldots & \phi_{j1m'} \\
\phi_{i1j} & \phi_{i1j} & 1 & \ldots & \phi_{i1im} & \phi_{i1j1} & \ldots & \phi_{i1jm'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
\phi_{imj} & (1 - \phi_{ij}^2) & \phi_{imj} & \phi_{imj1} & \ldots & 1 & \phi_{imjm} & \phi_{imjm'} \\
0 & \phi_{k1j} & \phi_{k1j} & \ldots & \phi_{k1im} & 1 & \phi_{k1jm} & \phi_{k1jm'} \\
0 & \phi_{j1j} & \phi_{j1j} & \ldots & \phi_{j1im} & \phi_{j1jm} & 1 & \ldots & \phi_{j1jm'} \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \phi_{jm1j} & \phi_{jm1j} & \ldots & \phi_{jm1im} & \phi_{jm1jm} & \phi_{jm1jm'} & 1 \\
\end{vmatrix}
\]

\[
= (1 - \phi_{ij}^2) \left( f_{ii} + \sum_{n=1}^{m} \phi_{in} f_{in} \right)
\]

\[
= (1 - \phi_{ij}^2) \left( f_{ii} - \sum_{n=1}^{m} \phi_{in}^2 \frac{|\Phi|}{1 - \phi_{ij}^2} \right).
\]

where the last equality comes from (21). Therefore,

\[
f_{ii} = \left( \frac{1}{1 - \phi_{ij}^2} + \sum_{n=1}^{m} \frac{\phi_{in}^2}{1 - \phi_{ij}^2} \right) |\Phi|.
\]

(22)

(iv) We know from (16) that the spatial equilibrium is given by

\[
\lambda^* = \sum_l f_{kl} \theta_l.
\]

Since \(|\Phi|\) is common to the numerator and denominator of \(f_{kl}/\sum_n f_{nl}\), conditions (20), (21) and (22) show that this coefficient is a function of \(\phi_{kl}\) and \(\phi_{nl}\) only when \((k,l) \in E\) and \((n,l) \in E\). However, since by assumption \((k,i) \notin E\) and \((k,j) \notin E\), the coefficient does not contain \(\phi_{ij}\). Hence, \(\partial \lambda^*/\partial r_{ij} = 0\), which proves the second part of the lemma.

(v) To prove the first part of the lemma, note that from condition (20), \(\lambda^*_n\) reduces to

\[
\lambda^*_n = \frac{f_{ni} \theta_i + \sum_{j \neq i} f_{ij} \theta_j}{f_{ii} + \sum_{j \neq i} f_{ij}}.
\]
Using conditions (21) and (22), we get

\[
\frac{f_{ii} \theta_i}{f_{ii} + f_{ij} + \sum_{h=1}^{m} f_{ih}} = \frac{-\phi_{inn} \theta_i}{1 - \phi_{inn}} = \frac{\phi_{ij}}{1 - \phi_{ijn}} - \frac{\phi_{ij}}{1 - \phi_{ijn}^{2}} - \sum_{h=1}^{m} \frac{\phi_{ijn}}{1 - \phi_{ijn}^{2}},
\]

which establishes the first part of the lemma.

**Appendix 3: Proof of Proposition 4**

(i) From (17), we know that the indirect utility \( U^*_i \) is inversely related to

\[
\varphi_i = \frac{\sum_{j=1}^{M} f_{ij}}{|\Phi|} = \frac{f_{ii} + f_{ij} + \sum_{n=1}^{m} f_{in}}{|\Phi|}.
\]

Using conditions (21) and (22), we have

\[
\varphi_i = \left[ 1 + (1 - \phi_{ij}) \sum_{n=1}^{m} \frac{\phi_{ijn}^{2}}{1 - \phi_{ijn}^{2}} \right] \frac{1}{1 - \phi_{ij}^{2}} - \sum_{n=1}^{m} \frac{\phi_{ijn}}{1 - \phi_{ijn}^{2}} - \phi_{ij}
\]

\[
= \frac{1}{1 + \phi_{ij}^{2}} - \sum_{n=1}^{m} \frac{\phi_{ijn}}{1 + \phi_{ijn}^{2}}.
\]

Hence,

\[
\frac{\partial \varphi_i^{-1}}{\partial \phi_{ij}} = \frac{1}{1 - (1 + \phi_{ij}) \sum_{n=1}^{m} \frac{\phi_{ijn}}{1 + \phi_{ijn}^{2}}} > 0,
\]

which implies that \( \partial U^*_i / \partial \phi_{ij} > 0 \), and hence \( \partial U^*_k / \partial r_{ij} < 0 \) for all \( k \neq i, j \).

(ii) Since \( \varphi_i \) is a function of \( \phi_{il} \) \( (l = j, i_1, \ldots, i_n) \) only, we readily get \( \partial U^*_k / \partial \phi_{ij} = \partial U^*_k / \partial r_{ij} = 0 \) for all \( k \neq i, j \).
Figure 1: Graph economy with 9 countries and 10 edges