Abstract

We study market liberalisation under imperfect competition in the presence of price effects. For this purpose, we build a three-country model of international trade under monopolistic competition. The neighbouring effect translates how the size effect propagates across countries. When some country increases in size, its relative wage increases, as well as that in a small and near country, while that in a large and distant country falls. We also show that a preferential trade agreement increases the relative wage, the welfare, and the terms-of-trade in the partner countries, where the integration effect dominates, while lowering those in the third country.

**JEL Classification:** F12, F15, R13

**Key-words:** monopolistic competition, market size effect, preferential trade agreement
In imperfect competition models of international trade, the existence of a costlessly traded homogeneous good sector has often been assumed, especially when dealing with multi-country models. This leads to Factor Price Equalisation (FPE) across countries, which significantly simplifies the analysis.\(^1\) In that context, markets integrate via the relocation of firms and workers across countries, see Krugman (1980), Baldwin \textit{et al.} (2003), Behrens \textit{et al.} (2007), Venables (1987), or Ossa (2011). By focusing on the consequences of production shifting and of the relocation of industry, that line of research abstracts completely from any price effect present during the liberalisation process. In particular, it assumes away terms-of-trade considerations and their impact on welfare. Moreover, in the real world, FPE does not hold, even between developed countries.\(^2\)

In this paper, we address the consequences of market liberalisation in a framework dealing with size, neighbouring, price, and integration effects. For this purpose, following Venables (1987) and Ossa (2011), among others, we build on Krugman’s (1980) new trade theory to construct a three-country model of international trade under monopolistic competition. In contrast to the existing literature, we relax the assumption of FPE by removing the costlessly traded good sector, so that prices and wages are endogenous, and price effects are included into the analysis. As our framework deals with an arbitrary trade cost structure between countries, our results go beyond the analysis of specific examples such as the symmetric or the hub-and-spoke configurations studied by Puga and Venables (1997). Moreover, unlike in Ossa (2011), no trade restriction is placed between countries. Hence, our model deals with general trade patterns allowing for asymmetries in country sizes and trade costs.

\(^1\)FPE is a direct consequence of costless trade in the constant returns sector. Davis (1998) shows how costly trade in both the constant and the increasing returns sectors substantially alters the equilibrium outcome.

\(^2\)The counterfactual prediction of FPE could be avoided by allowing for different productivities across countries in the homogeneous good sector. However, these productivities would not be endogenous.
The aim of this paper is twofold. First, we look at the role of country size on wages and welfare. Second, we analyse the impact of Preferential Trade Agreements (PTAs) on the partner countries and the left-out country. In this study, we consider the simplest setting allowing for third country effects and terms-of-trade movements, namely a three-country Krugman model (1980), for which a unique equilibrium is shown to exist.

The first set of results relate to size and neighbouring effects. When some country increases in size, its relative wage increases, as well as that in a small and near country, while that in a large and distant country falls. This result extends the size effect emphasized by Krugman (1980) in the case of two countries by introducing a neighbouring effect, which translates how the size effect propagates across countries. The increase in market potential is larger in a neighbouring country than in a distant one. In terms of welfare, all countries gain from the increase in some country size because world production and consumption end up increasing.

The second set of results relate to the consequences of PTAs. When some countries engage in a PTA, the integration effect induces relative prices including relative wages to increase in the integrating area. By raising the export price in the partner countries, the effect of a PTA is to improve the terms-of-trade in the integrating area, while lowering that in the excluded country. While a PTA is beneficial to the partner countries in utility terms, it is always detrimental to the third country because the latter one does not benefit from the integration effect and is exposed to a negative price effect: it has to import goods produced at higher costs in the integrating area.

Third, a natural question is whether our PTAs results could hold in $N$-country environments. We provide several examples for which this is the case.

A key property of Krugman’s (1980) framework is that shocks affecting labour endowments and trade costs transmit either through terms-of-trade or production relocation effects. In the
absence of production relocation effects, Krugman’s model behaves pretty much like a Ricardian or an Armington model, though they differ in their microfoundations. This similarity is now well understood, see Arkolakis et al. (2012). That class of trade models also includes Eaton and Kortum (2002) and Melitz (2003) models. Many recent papers have built on that Armington structure. For instance, Chaney (2008) has used Krugman’s framework with asymmetric countries and trade costs. However, his framework is different from ours for the following reason. So as to deal with firm heterogeneity and to focus on extensive versus intensive margins of trade, he assumed the presence of an homogenous good which is freely traded. By using that good as the numeraire, wages are pinned down by production parameters, instead of being determined by general equilibrium conditions as is the case in our paper. Hsieh and Ossa (2011) consider a multi-country Melitz model with multiple industries. Their objective is to estimate empirically the effect of productivity shocks, not of size or trade cost shocks, on welfare. Moreover, in order to account for general equilibrium adjustments, they rely on a numerical approximation procedure.

In the context of an Armington structure, Arkolakis et al. (2012) have shown that the change in welfare associated with any foreign shock can be calculated from the change in the share of domestic expenditure. This important result provides a method for estimating welfare gains irrespective of the source of foreign shocks. However, their approach provides no indication on how these shares of domestic expenditures could respond to these various shocks in a general equilibrium environment. In contrast to their analysis, our work identifies the effect all types of shocks including domestic ones on countries’ welfare in the context of Krugman’s framework. By doing so, we also provide the corresponding implicit changes in the shares of domestic expenditure used by Arkolakis et al. (2012).

The role of trade policy intervention in imperfectly competitive markets is analysed in Bag-
well and Staiger (2009): the rationale for a trade agreement is to overcome the inefficiency related to the terms-of-trade externality. Here the terms-of-trade gain provides a strong incentive for countries to engage in bilateral trade agreements. This result is in line with that obtained in standard neoclassical trade models, e.g. Maggi and Rodriguez-Clare (1998), and other new trade models in the presence of a freely traded homogeneous good, see the trade policy implications derived by Puga and Venables (1997), as well as the model by Ossa (2011) where the third country trades with one partner country only. However, in general, when trade is not restricted so that each country trades with any other one, a PTA may hurt some partner country in terms of welfare under FPE. This has been shown to happen when the hub effect is large enough, see Behrens et al. (2007). Here, in contrast to the models relying on FPE, the integration effect always dominates the price effect irrespective of country sizes and of the spatial distribution of resources across countries, so that welfare always increases in the integrating area.

Under FPE, falling trade costs between countries concluding the PTA is accompanied by the relocation of firms to the PTA partners, while it does not lead to terms-of-trade movements. The relocation of firms from the third country to the PTA partners implies a worse access of the third country to the manufacturing varieties. In contrast, in this paper, another rationale for the lower utility in the third country is provided. The falling trade costs between PTA partners raise prices and wages in the integrating area relative to the price level in the third country. As a result, consumers in the left-out country suffer a terms-of-trade loss because they have to import varieties produced at higher costs in the partner countries, which lowers their welfare.

The literature on Free Trade Agreements (FTAs) has studied a wide range of determinants affecting the incentives of countries to enter an FTA. By relying on a three-continent model with two countries on each continent, Baier and Bergstrand (2004) developed a computational general approach to determine empirically how partner countries’ characteristics (e.g. the distance
between them, their market size, or their factor endowments) and also other factors (e.g. their remoteness from other countries) affect the welfare of member countries. In that respect, the scope of our paper is more limited than theirs. In order to keep their sophisticated model tractable, asymmetries in transport costs are neglected. Because Krugman’s structure is more stylized, we are able to provide a formal positive analysis for both member and non-member countries allowing for general asymmetries including those in trade costs. Egger and Larch (2008) analysed empirically how a pre-existing PTA affects the incentive to form a new PTA. More generally, Baldwin and Jaimovich (2012) analyse spreading regionalism (i.e., the contagion nature of FTAs) based on a multi-country framework with monopolistic competition under the assumption of FPE. Our contribution with respect this -mainly empirical- literature on FTAs is theoretical. Our analysis provides clear directions for price and welfare changes allowing for third-country effects (due to country or trade cost asymmetries) and terms-of-trade movements. According to us, this is a step forward in better understanding general third country effects (among which FTA interdependence is an important particular case) in international trade. Our model derives implications not only for PTA members but also for left-out countries.

The remainder of the paper is organised as follows. The three-country model of international trade under imperfect competition is introduced in Section 1. Section 2 presents some preliminary results of the model. In Section 3, we analyse the role of country size and the impact of a PTA on wages and welfare. Section 4 extends the PTA results to several \( N \)-country economies. Section 5 concludes.

1. The Model

The economy consists of three countries and a manufacturing sector producing a differentiated good. The mass of immobile workers in country \( i \) is denoted by \( L_i \). Without loss of generality, we assume that the total mass of workers is \( \sum_{i=1}^{3} L_i = 1 \).
The utility of an individual in country $i$ is given by Dixit-Stiglitz preferences

$$U_i = \left( \sum_{j=1}^{3} \int_{v \in V_j} q_{ji}(v)^{\frac{\sigma-1}{\sigma}} \, dv \right)^{\frac{\sigma}{\sigma-1}},$$

where $q_{ji}(v)$ is the amount of variety $v$ produced in country $j$ and consumed in country $i$, $V_j$ is the set of varieties produced in country $j$, and $\sigma > 1$ is the elasticity of substitution between any two varieties. The budget constraint of a worker in country $i$ earning a wage $w_i$ is given by

$$\sum_j \int_{v \in V_j} p_{ji}(v)q_{ji}(v) \, dv = w_i,$$

where $p_{ji}(v)$ is the delivery price of variety $v$ produced in country $j$ and consumed in country $i$.

In order to simplify the notation, we drop the variety label $v$ from now on. The maximisation of utility (1) subject to the budget constraint (2) yields the following worker’s demand in country $i$ for a variety produced in country $j$:

$$q_{ji} = \frac{p_{ji}^{-\sigma}}{P_i^{1-\sigma}} w_i$$

with the price index $P_i$ in country $i$ given by

$$P_i = \left( \frac{1}{\sum_k n_k p_{ki}^{1-\sigma}} \right)^{\frac{1}{1-\sigma}},$$

where $n_k$ is the mass of firms located in country $k$.

Assuming iceberg trade costs, $\tau_{ij} > 1$ units of a variety have to be shipped from country $i$ for one unit of that variety to reach country $j (\neq i)$. We also assume that these trade costs are symmetric $\tau_{ij} = \tau_{ji}$ and $\tau_{ii} = 1$.

The production technology requires a fixed and a constant marginal labour requirements, labeled $f$ and $c$ respectively.\textsuperscript{3} In order to satisfy the demand $q_{ij}L_j$ in country $j$, each firm in

\textsuperscript{3}Because immobile labour is the only production factor, the equilibrium number of firms in each country turns out to be constant. As a result, there is no production relocation effect à la Krugman (1980).
country \(i\) has to produce \(\tau_{ij} q_{ij} L_j\) units. Thus, the profit of a firm in country \(i\) is given by

\[
\pi_i = \left( \sum_j p_{ij} q_{ij} L_j \right) - w_i \left( f + c \sum_j \tau_{ij} q_{ij} L_j \right). \tag{5}
\]

By plugging the worker’s demand (3) into expression (5), profit maximisation with respect to prices yields

\[
p_{ij} = \frac{\sigma c \tau_{ij}}{\sigma - 1} w_i. \tag{6}
\]

By assuming free entry and exit of manufacturing firms, profit (5) is zero. Given that \(p_{ij} = p_{ii} \tau_{ij}\), we have

\[
\left(p_{ii} - cw_i\right) \sum_j \tau_{ij} q_{ij} L_j = w_i f. \tag{7}
\]

Because labour inputs are given by the second bracketed terms in expression (5), the labour market clearing condition is

\[
n_i \left(f + c \sum_j \tau_{ij} q_{ij} L_j \right) = L_i. \tag{8}
\]

Using relations (6), (7), and (8), the equilibrium number of firms is proportional to the number of workers as follows:

\[
n_i = \frac{L_i}{\sigma f} \tag{9}
\]

By substituting relations (6) and (9) into the profit expression (5), we have

\[
\sum_j \frac{\tau_{ij}^{1-\sigma} L_j w_j}{\sum_k \tau_{kj}^{1-\sigma} L_k w_k^{1-\sigma}} = w_i^\sigma, \quad \text{for } i = 1, 2, 3. \tag{10}
\]

Wages \(w_i\) are determined by these three equilibrium conditions. By Walras’ law, one of these conditions is redundant, so that labour in some country can serve as numéraire.

The equilibrium utility in country \(i\) is given by

\[
U_i^* = \frac{w_i}{P_i} = \frac{w_i}{\frac{\sigma c}{\sigma - 1} \left( \frac{1}{\sigma f} \sum_k \tau_{ki}^{1-\sigma} L_k w_k^{1-\sigma} \right)^{\frac{1}{1-\sigma}}}, \tag{11}
\]

where wages are evaluated at equilibrium (10).
2. Preliminary Results

First of all, as stated in previous section already, we assume that shipping a manufacturing variety from one country to another is costly, so that we exclude the case of costless trade $\tau_{ij} = 1$, $i \neq j$.

**ASSUMPTION 1** *For any distinct $i, j, k \in \{1, 2, 3\}$, $\tau_{ij} > 1$.*

Assumption 1 implies costly international trade and excludes perfect integration between countries. This assumption is in no way restrictive given that otherwise, the number of countries would reduce to two or less.

We now show that our model is general enough to encompass both the direct- and the indirect-shipping of goods. While $\tau_{ij}$ represents the direct-trade cost between countries $i$ and $j$, the product $\tau_{ik}\tau_{jk}$ corresponds to the trade cost between these countries when the good is shipped via country $k$. The former trade cost is more costly than the latter one if, for instance, a very high tariff is imposed between countries $i$ and $j$. In this case, direct trade is more costly than trade via a third country. However, should this arises, it could only be so for one pair of countries, not more. To see this, denote the largest trade cost by $\tau_{ij}$ so that $\tau_{kl} \leq \tau_{ij}$, $\forall k, l$, where $i \neq j$ and $k \neq l$. In particular, this implies that $\tau_{ik} \leq \tau_{ij} < \tau_{ij}\tau_{jk}$ and $\tau_{jk} \leq \tau_{ij} < \tau_{ij}\tau_{ik}$, for $k$ different from $i$ and $j$, which means that at least two triangle inequalities are always satisfied. In other words, direct trade cannot be more costly than indirect trade for two pairs of countries. This is because the presence of two very costly direct-shipping routes makes indirect-shipping simply not possible in our three-country model, as it would involve using two non-costly shipping routes.
Two cases may arise:

(i) \( \tau_{ik} < \tau_{ij} \tau_{jk}, \ \tau_{ij} < \tau_{ik} \tau_{jk}, \ \text{and} \ \tau_{jk} < \tau_{ij} \tau_{ik} \)

(ii) \( \tau_{ik} \geq \tau_{ij} \tau_{jk}, \ \tau_{ij} < \tau_{ik} \tau_{jk}, \ \text{and} \ \tau_{jk} < \tau_{ij} \tau_{ik}. \)

In case (i), direct trade is less costly than trade via a third country for any pair of countries, so that it involves direct-shipping only, and the triangle inequality always holds. An example of this situation is when trade costs correspond to distance-related transport costs.

In case (ii), direct trade is more costly than trade via a third country for one pair of countries \((i, k)\) and the triangle inequality is violated for that pair of countries. In this latter case, we will assume that firms transport goods from country \(i\) to country \(k\) via country \(j\) rather than directly so that the effective trade cost \( \tau_{ik}^{'} \) is given by \( \tau_{ij} \tau_{jk} \). For example, if tariffs between countries \(i\) and \(k\) are very high, then firms will avoid direct trade by shipping goods via the third country \(j\) in order to reduce trade costs. Note that case (ii) is more likely to occur in international trade than in interregional trade because within a country trade costs increase in the geographical distance.

To make our model as general as possible and so as to encompass the possibility of indirect-shipping, we define the effective trade cost \( \tau_{ik}^{'} = \min\{\tau_{ik}, \tau_{ij} \tau_{jk}\} \). This definition can be rewritten in terms of the freeness of trade \( \phi_{ij} = \tau_{ij}^{-\sigma} \in (0, 1] \) between countries \(i\) and \(j\) in the following way. For any distinct \(i, j, k \in \{1, 2, 3\}\), the effective freeness of trade is given by \( \phi_{ik}^{'} = \max\{\phi_{ik}, \phi_{ij} \phi_{jk}\} \). From Assumption 1, without loss of generality, we can set

\[
\phi_{23}^{'} = \max\{\phi_{23}, \phi_{12} \phi_{13}\}, \ \phi_{12}^{'} = \phi_{12} > \phi_{13} \phi_{23}, \ \text{and} \ \phi_{13}^{'} = \phi_{13} > \phi_{12} \phi_{23}, \tag{12}
\]

From now on, in order to simplify the writing, we drop the ‘ notation and define the freeness of
trade matrix $\Phi_3$ by

$$\Phi_3 = \begin{pmatrix} 1 & \phi_{12} & \phi_{13} \\ \phi_{12} & 1 & \phi_{23} \\ \phi_{13} & \phi_{23} & 1 \end{pmatrix}.$$  

We now propose a useful way of restating the wage and utility equations (10) and (11).

**LEMMA 1** *Wages and utilities are determined by*

$$v_i = \sum_{j=1}^{3} \phi_{ij} L_j v_j^{-\varepsilon}, \quad i = 1, 2, 3, \quad (13)$$

where $v_i$ stands either for $w_i^\sigma$ or $\gamma U_i^{\frac{\sigma (\sigma - 1)}{2 \sigma - 1}}$, with $\varepsilon = (\sigma - 1)/\sigma$ and $\gamma = (\sigma f)^{\frac{\sigma}{2 \sigma - 1}} [\sigma c/(\sigma - 1)]^{\frac{\sigma (\sigma - 1)}{2 \sigma - 1}}$.

**Proof.** This result has been proved by Tabata *et al.* (2013) in the case of an infinite-dimensional economy. For the sake of clarity, we now show the validity of the proposed decomposition (13).

By using the freeness of trade notation, wage equations (10) can be rewritten as

$$w_i^\sigma = \sum_j \frac{\phi_{ij} L_j w_j}{w_i^\sigma}, \quad i = 1, 2, 3.$$

By making the following substitution

$$w_j^\sigma = C \sum_k \phi_{jk} L_k w_k^{1-\sigma},$$

where $C$ is some positive constant, we get

$$w_i^\sigma = \sum_j \frac{\phi_{ij} L_j w_j}{w_j^\sigma/C} = C \sum_j \phi_{ij} L_j w_j^{1-\sigma}.$$

By defining $v_i = w_i^\sigma$, we have

$$v_i = C \sum_{j=1}^{3} \phi_{ij} L_j v_j^{-\varepsilon}, \quad (14)$$

where $\varepsilon = (\sigma - 1)/\sigma \in (0, 1)$. 
The uniqueness of the decomposition (14) follows directly from Tabata et al. (2013) by adapting their argument to the case of a finite-dimensional economy.

Moreover, using $U_i = w_i / P_i$ yields

$$w_i^\sigma = \gamma C^{\sigma - 1} U_i^{\sigma - 1},$$

where $\gamma = (\sigma f)^{\sigma - 1} [\sigma c / (\sigma - 1)]^{\sigma - 1}$. This means that the equilibrium utility in country $i$ depends on the local wage only. As a consequence, the equilibrium equations (14) also describe utilities by making the substitution $v_i = \gamma U_i^{\sigma - 1}$ and considering $C = 1$. In the rest of the paper, we normalize $C = 1$. ■

To analyse the equilibrium conditions (13), it is convenient to define the map $F$ as follows

$$F_i(v) \equiv v_i - \sum_{j=1}^3 \phi_{ij} z_j = 0, \quad i = 1, 2, 3,$$

where $z_i$ denotes $L_i v_i^{-x}$. The Jacobian matrix $D_v F$ of these equations is given by

$$D_v F = \begin{pmatrix} 1 + \varepsilon x_1 & \varepsilon \phi_{12} x_2 & \varepsilon \phi_{13} x_3 \\ \varepsilon \phi_{12} x_1 & 1 + \varepsilon x_2 & \varepsilon \phi_{23} x_3 \\ \varepsilon \phi_{13} x_1 & \varepsilon \phi_{23} x_2 & 1 + \varepsilon x_3 \end{pmatrix},$$

where $x_i$ denotes $z_i / v_i$.

The freeness of trade matrix and the Jacobian of the equilibrium map have a positive determinant.

**Lemma 2** $\det \Phi_3 > 0$ and $\det(D_v F) > 0$.

**Proof.** First, notice that the determinant of the freeness of trade matrix can be written as

$$\det \Phi_3 = 1 - \phi_{12}^2 - \phi_{13}^2 - \phi_{23}^2 + 2\phi_{12}\phi_{13}\phi_{23}$$

$$= (\phi_{12} - \phi_{13}\phi_{23})(1 - \phi_{12}) + (\phi_{13} - \phi_{12}\phi_{23})(1 - \phi_{13})$$

$$+ (\phi_{23} - \phi_{12}\phi_{13})(1 - \phi_{23}) + (1 - \phi_{12})(1 - \phi_{13})(1 - \phi_{23}).$$
Then, by using the triangle inequalities (12), we get \( \det \Phi_3 > 0 \).

Second, by using the expression (16) of the Jacobian matrix \( D_v F \), we have

\[
\det(D_v F) = 1 + \varepsilon(x_1 + x_2 + x_3) + \varepsilon^2[(1 - \phi_{12}^2)x_1x_2 + (1 - \phi_{13}^2)x_1x_3 + (1 - \phi_{23}^2)x_2x_3]
\]

\[
+ \varepsilon^3x_1x_2x_3 \det \Phi_3,
\]

which implies that \( \det(D_v F) > 0 \) given that \( \det \Phi_3 > 0 \). \( \blacksquare \)

We now address the existence and uniqueness of equilibrium.

**PROPOSITION 1** The model admits a unique equilibrium.

**Proof.** First, rewrite \( F \) as \( I - H \) so that the map \( H \) is defined by

\[
H_i(v) \equiv \sum_{j=1}^{N} \phi_{ij} L_j v_j^{-\varepsilon}, \quad i = 1, 2, 3.
\]

Let \( \phi = \min \phi_{ij} \) and observe that \( H \) maps \( D = [\phi^{1-\varepsilon^2}, \phi^{-\frac{\varepsilon}{1-\varepsilon^2}}] \times [\phi^{1-\varepsilon^2}, \phi^{-\frac{\varepsilon}{1-\varepsilon^2}}] \times [\phi^{1-\varepsilon^2}, \phi^{-\frac{\varepsilon}{1-\varepsilon^2}}] \subset \mathbb{R}_+^3 \) into itself

\[
H_i(v) \geq \phi \sum_j L_j v_j^{-\varepsilon} \geq \phi \sum_j L_j \phi^{1-\varepsilon^2} = \phi^{1-\varepsilon^2}
\]

\[
H_i(v) \leq \sum_j L_j v_j^{-\varepsilon} \leq \sum_j L_j \phi^{-\frac{\varepsilon}{1-\varepsilon^2}} = \phi^{-\frac{\varepsilon}{1-\varepsilon^2}},
\]

where the total labour constraint (\( \sum_j L_j = 1 \)) has been used. This shows that an equilibrium exists by Brouwer fixed-point theorem. Note that this first part of the proof is similar to the method used by Tabata et al. (2013) for an economy with an infinite number of locations.

Second, the uniqueness of equilibrium follows from an homotopy argument. For this, consider the parametric dependence of \( v \) on \( \varepsilon \) and define the homotopy \( K(v, \varepsilon) = F(v) \) with \( \varepsilon \geq 0 \). We are interested in the structure of the equilibrium set \( v(\varepsilon) : K(v, \varepsilon) = 0, \forall \varepsilon \geq 0 \). By the first part of the proof, a solution \( v(\varepsilon) \) is known to exist, \( \forall \varepsilon \). We now study the behaviour of \( v(\varepsilon) \) by considering its graph in the plane \( (v, \varepsilon) \). When \( \varepsilon = 0 \), the solution \( v \) is unique given the
expression of the equilibrium map \( v_i(0) = L_i + \sum_{j \neq i} \phi_{ij} L_j \). The local behavior of \( v(\varepsilon) \) around \( \varepsilon = 0 \) is given by the implicit function theorem

\[
D\varepsilon v|_{\varepsilon=0} = -(DvK)^{-1}D\varepsilon K|_{\varepsilon=0}.
\]

By Lemma 2, \( \det DvK|_{\varepsilon=0} = \det DvF|_{\varepsilon=0} \neq 0 \), which means that the homotopy is regular when \( \varepsilon = 0 \) (i.e. locally, \( v \) can be seen as a function of \( \varepsilon \)). Note that the homotopy remains regular as \( \varepsilon \) increases away from zero as \( \det DvK|_{\varepsilon=0} = \det DvF|_{\varepsilon=0} \neq 0 \), which ensures that the curve \( v(\varepsilon) \) defines a smooth path starting at \( \varepsilon = 0 \), thereby preventing the path to bifurcate or to exhibit an horizontal tangency in the plane \((v, \varepsilon)\). Regarding the global behaviour of \( v(\varepsilon) \), the path emanating from \( \varepsilon = 0 \) is necessarily unique. This is because any other path would intersect \( \varepsilon = 0 \), which would imply multiplicity of equilibria when \( \varepsilon = 0 \).

While equations (15) will be used to study utilities, we still need to derive equations helpful to analyse relative wages across countries. By Walras’ law, variables \( v_1 \) and \( v_2 \) can be expressed relative to \( v_3 \) by normalizing \( v_3 = 1 \):

\[
\begin{align*}
G_1(v_1, v_2) &\equiv v_1(\phi_{13}z_1 + \phi_{23}z_2 + L_3) - (z_1 + \phi_{12}z_2 + \phi_{13}L_3) \\
G_2(v_1, v_2) &\equiv v_2(\phi_{13}z_1 + \phi_{23}z_2 + L_3) - (\phi_{12}z_1 + z_2 + \phi_{23}L_3),
\end{align*}
\]

(17)

where \( z_i \) still denotes \( L_i v_i^{-\varepsilon} \).

We now show that admissible relative \( v \)'s belong to the triangle \( \Delta \) defined by the sides \( l_i(v_1, v_2, 1) = 0 \) for \( i = 1, 2, 3 \) in the plane \((v_1, v_2) = (w_1^\sigma, w_2^\sigma)\) in Figure 1. \( l_i(v_1, v_2, 1) \) is obtained by by setting \( L_i = 0 \) and eliminating \( z_j \) and \( z_k \) from (13) for distinct \( i, j, k \). The three corners of \( \Delta \) correspond to the cases \( L_i = 1, \forall i \). This is because side \((l_i = 0)\) corresponds to \( L_i = 0 \) meaning that the corner \((l_j = 0) \cap (l_k = 0)\) corresponds to \( L_i = 1 \) for distinct \( i, j, k \). For instance, when \( L_1 = 1, (v_1, v_2) = (\phi_{13}^{-1}, \phi_{12}/\phi_{13}) \), which corresponds to the corner opposite to side \( l_1 = 0 \). Similarly, when \( L_2 = 1, (v_1, v_2) = (\phi_{12}/\phi_{23}, \phi_{23}^{-1}) \), which corresponds to the corner opposite to side \( l_2 = 0 \). In the rest of the paper, we focus on the case \( L_i > 0, \forall i \). Otherwise
the number of countries would reduce to two or less. When \( L_i > 0, \forall i \), it must be that \( L_i > 0, \forall i \), meaning that admissible \((v_1, v_2)\)'s belong to the interior of triangle \( \Delta \).

![Figure 1: Admissible triangle \( \Delta \)](image-url)

3. Comparative Statics

First, we examine the general equilibrium impacts of an exogenous increase in country size on utilities and relative wages. We then study the consequences of a PTA by considering an exogenous increase in the freeness of trade between the countries concluding the PTA.

3.1. Size effect

**PROPOSITION 2** For any distinct \( i, j, k \in \{1, 2, 3\} \),

(i) \( \left( \frac{w_j}{w_i} \right) / dL_j > 0 \), and

(ii) \( \left( \frac{w_k}{w_i} \right) / dL_j \geq 0 \) if \( w_k/w_i \leq \left( \frac{\phi_{jk}}{\phi_{ij}} \right)^{1/\sigma} \).

The proof is contained in Appendix A.

Proposition 2(i) implies that the larger a country, the higher its relative wage. This result corresponds to the size effect emphasized by Krugman (1980, p. 954) in the case of two countries. When the size of the local market increases, local firms face lower average transportation costs.
In equilibrium, that competitive advantage is offset by higher relative local wages. Proposition 2(ii) shows that the size effect may affect neighbouring countries in a multi-country setting. This neighbouring effect says that a country size increase will tend to increase (resp. lower) relative wages in other countries provided that they are low enough (resp. high enough). To interpret this result, observe first that \( \frac{w_k}{w_i} < \left( \frac{\phi_{jk}}{\phi_{ij}} \right)^{1/\sigma} \) as long as country \( k \) is sufficiently small and the freeness of trade between countries \( j \) and \( k \) sufficiently high. This means that an exogenous increase in the population of some country tends to increase (resp. lower) the relative wage in a small and near country (resp. a large and distant country). The neighbouring effect translates the size effect propagates across countries: when some country increases in size, the increase in market potential is larger in a neighbouring country than in a remote one. Also, this impact is larger on a small country than on a large one as smaller countries are more sensitive to foreign shocks.

**PROPOSITION 3** \( \frac{dU_i}{dL_j} > 0, \forall i \text{ and } j. \)

The proof is contained in Appendix B.

An increase in the local labour force is beneficial to all countries because world consumption ends up increasing. While the larger number of manufacturing workers leads to more local varieties and to higher output, the increase in demand raises the relative wage of local workers, which in turn makes local goods more expensive. Though the impact on relative wages in other countries may be positive or negative (depending on the sign of the neighbouring effect), Proposition 3 shows that overall, all countries gain in terms of welfare when some country increases in size.

We now look at the corresponding impact on trade flows. Let \( \lambda_{ij} \) denote the share of country \( j \)'s total expenditure on goods imported from country \( i \).
COROLLARY 1 (i) \( d\lambda_{ij}/dL_i > 0 \), \( \forall i \) and \( j \), and  
(ii) \( d\lambda_{kj}/dL_i < 0 \), \( \forall j \) and \( k \neq i \).

The proof is contained in Appendix B.

So, when some country increases in size, all the import shares from that country increase while those from the other countries fall. Arkolakis et al. (2012) determine how welfare changes can be inferred from changes in the share of domestic expenditures. Our results complement their result by determining how trade shares and welfare respond simultaneously to country size shocks in Krugman’s model.

3.2. The Impact of a PTA

In this section, we consider the scenario where countries \( j \) and \( k \) engage in a PTA. Market integration is studied by investigating the impact of an increase in the freeness of trade between PTA partners on relative wages and welfare. By neglecting the source of potential tariff revenues for partner countries, our approach follows Venables (1987), Behrens et al. (2007), and Ossa (2011).

PROPOSITION 4 For any distinct \( i, j, k \in \{1, 2, 3\} \),

\[
d(w_j/w_i)/d\phi_{jk} > 0.
\]

The proof is contained in Appendix C.

Proposition 4 states that a PTA between two countries via a reduction in their mutual trade cost increases their wages relative to that of the third country. The integration effect due to a better market access between PTA partners induces the price index in the integrating area to fall and local consumption to rise. However, because supply is fixed, the price effect leads prices and relative wages in the integrating area to rise so as to restore equilibrium. Because the export price is proportional to the wage in the export country (see expression (6)), this price
effect improves the terms-of-trade of the integrating area, while lowering that of the excluded country. The implication on welfare is derived in the following Proposition.

**PROPOSITION 5** For any distinct $i, j, k \in \{1, 2, 3\}$,

(i) $dU_j/d\phi_{jk} > 0$ and $dU_k/d\phi_{jk} > 0$.

(ii) $dU_i/d\phi_{jk} < 0$.

The proof is contained in Appendix D.

Proposition 5(i) is intuitive: the terms-of-trade gain provides a strong incentive for countries to engage in bilateral trade agreements. This result is similar to that obtained in other new trade models in the presence of a freely traded homogeneous good, see the trade policy implications derived in the symmetric and hub-and-spoke configurations by Puga and Venables (1997), as well as the model by Ossa (2011) where the third country trades with one partner country only. However, in general, when trade is not restricted so that each country trades with any other one, a PTA may hurt some partner country in terms of welfare under FPE. This has been shown to happen when the hub effect is large enough, see the example with $dU_j/d\phi_{jk} < 0$ provided by Behrens et al. (2007, p. 637). In that case, although the two countries engaging in a PTA have a better market access and attract firms overall, firms in the smaller country move to the larger one (Baldwin and Robert-Nicoud, 2000), which reduces the welfare in the smaller country concluding the PTA. Here, in contrast to the models relying on FPE, the integration effect always dominates the price effect irrespective of country sizes and of the spatial distribution of resources across countries, so that welfare always increases in the integrating area.

Proposition 5(ii) is another important finding of this paper. It states that while a PTA is beneficial to PTA partners, it is always detrimental to the third country. This result is in line with those obtained under FPE, see Puga and Venables (1997), Behrens et al. (2007, Proposition 3), and Ossa (2011, Proposition 3). However, here, another rationale for the lower utility in the
third country is provided. Under FPE, the falling trade costs between countries concluding the PTA are accompanied by the relocation of firms to PTA partners, while it does not lead to terms-of-trade movements. In other words, the relocation of firms from the third country to PTA partners implies a worse access of the third country to the manufacturing varieties. In contrast, in this paper, the falling trade costs \( \phi_{jk} \) between PTA partners raise prices and relative wages in the integrating countries \( j \) and \( k \), with respect to the price level in the third country \( i \) (see Proposition 4). As a result, consumers in the third country do not benefit from the integration effect and are exposed to a negative price effect. They suffer a terms-of-trade loss because they have to import the varieties produced at higher costs in the partner countries, which lowers their welfare.

4. Some Extensions to \( N \) Countries

In this section, we extend our PTA results to several \( N \)-country economies.

We first consider the case of PTA partners \( 1 \) and \( 2 \) trading symmetrically with \( N - 2 \) symmetric third countries. Country sizes are given by \( L_1, L_2, L_3 = \ldots = L_N \), and the freeness of trade matrix \( \Phi \) by

\[
\Phi = \begin{pmatrix}
1 & \phi_{12} & \phi_{13} & \phi_{13} & \cdots & \phi_{13} \\
\phi_{12} & 1 & \phi_{23} & \phi_{23} & \cdots & \phi_{23} \\
\phi_{13} & \phi_{23} & 1 & \phi & \cdots & \phi \\
\phi_{13} & \phi_{23} & \phi & 1 & \ddots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\
\phi_{13} & \phi_{23} & \phi & \cdots & \phi & 1
\end{pmatrix},
\]

where \( \phi_{12} \) denotes the freeness of trade between countries \( 1 \) and \( 2 \), \( \phi_{13} \) (resp. \( \phi_{23} \)) that between country \( 1 \) (resp. country \( 2 \)) and any third country, and \( \phi \) that between any two third countries. To keep the example tractable, we assume \( \phi > \max\{\phi_{12}, \phi_{13}, \phi_{23}\} \). As third countries are of
identical size, the equilibrium equations (13) can be extended to

\[
\begin{pmatrix}
v_1 \\
v_2 \\
v_3
\end{pmatrix} = \begin{pmatrix}
z_1 + \phi_{12}z_2 + (N-2)\phi_{13}z_3 \\
\phi_{12}z_1 + z_2 + (N-2)\phi_{23}z_3 \\
\phi_{13}z_1 + \phi_{23}z_2 + [1 + (N-3)\phi]z_3
\end{pmatrix},
\]

(18)

where \( v_3 \) denotes \( v \) in any third country and \( z_i \) still denotes \( L_i v_i^{-\varepsilon-1} \), \( i = 1, 2, 3 \).

**PROPOSITION 6** Assume that PTA partners 1 and 2 trade symmetrically with \( N - 2 \) symmetric third countries. Then,

(i) \( dU_i/d\phi_{12} > 0 \), \( i = 1, 2 \)

(ii) \( dU_3/d\phi_{12} < 0 \) for any third country.

The proof is contained in Appendix E.

Our second extension explores a different special case. Here trade relationships between countries are assumed to be symmetric, while country sizes \( L_i \) may be asymmetric. The freeness of trade between any two countries is given by \( \phi \). The equilibrium equations (13) can then be written as

\[
v_i = z_i + \phi \sum_{j \neq i}^{N} z_j, \quad i = 1, \ldots, N,
\]

(19)

where \( z_i \) still denotes \( L_i v_i^{-\varepsilon-1} \).

Our last extension considers the case of low or high product differentiation between varieties \( (\sigma \to 1^+ \text{ or } \sigma \to \infty) \). No restriction is placed on country sizes nor on trade costs. The equilibrium equations (13) are now given by

\[
v_i = z_i + \sum_{j \neq i}^{N} \phi_{ij}z_j, \quad i = 1, \ldots, N.
\]

The PTA results corresponding to these last extensions are now summarized.
PROPOSITION 7 When trade relationships are symmetric or when product differentiation among varieties is either very low or very high,

(i) \( dU_i/d\phi_{ij} > 0 \)

(ii) \( dU_k/d\phi_{ij} < 0 \)

for any distinct countries \( i, j, \) and \( k \).

The proof is contained in Appendix F.

5. Conclusion

In this paper, we have built a three-country model of international trade under monopolistic competition. In contrast to the existing literature which relies on FPE across countries, our approach accounts for prices effects and terms-of-trade movements.

We have determined the role of country size on relative wages and welfare. When some country increases in size, its relative wage increases, as well as that in a small and near country, while that in a large and distant country falls. The size effect, emphasized by Krugman in the case of two countries, propagates across countries, giving rise to a neighbouring effect: the increase in market potential is larger in a neighbouring country than in a distant one. We have also determined the impact of a PTA on the participating countries and the left-out country. A PTA increases the relative wage, the welfare, and the terms-of-trade in the integrating area, while lowering those in the third country. These PTA results have been extended to a number of multi-country economies involving country size or trade cost asymmetries.

Further research aiming at a better understanding of third-country effects is still needed. In particular, quantifying the changes derived in the present work would be especially important as it could only enlarge the set of theoretically-informed PTAs propositions to be tested empirically. Also, extending our approach to multi-industry environments would be particularly relevant.
Appendix A: Proof of Proposition 2

It is sufficient to show the signs of \( \frac{d(w_2/w_3)}{dL_1} \) and \( \frac{d(w_2/w_3)}{dL_2} \) as the other results follow from a symmetric argument. Consider the relative wages equations (17). The corresponding Jacobian matrix \( D_v G \) is given by

\[
\begin{pmatrix}
L_3 + (1 - \varepsilon)\phi_{13}z_1 + \phi_{23}z_2 + \varepsilon x_1 & \varepsilon(\phi_{12} - \phi_{23}v_1)x_2 \\
\varepsilon(\phi_{12} - \phi_{13}v_2)x_1 & L_3 + \phi_{13}z_1 + (1 - \varepsilon)\phi_{23}z_2 + \varepsilon x_2
\end{pmatrix},
\]  

where \( x_i \) still denotes \( z_i/v_i \). We observe that \( \det(D_v G) > 0 \).

\[
\det(D_v G) = [L_3 + (1 - \varepsilon)\phi_{13}z_1 + \phi_{23}z_2 + \varepsilon x_1][L_3 + \phi_{13}z_1 + (1 - \varepsilon)\phi_{23}z_2 + \varepsilon x_2] - \varepsilon^2(\phi_{12} - \phi_{13}v_2)(\phi_{12} - \phi_{23}v_1)x_1x_2
\]

\[
> \phi_{23}z_1z_2 + \varepsilon x_1x_2 - \varepsilon^2(\phi_{12}^2x_1x_2 + \phi_{13}\phi_{23}v_1v_2x_1x_2)
\]

\[
= (1 - \varepsilon^2)\phi_{13}\phi_{23}z_1z_2 + \varepsilon^2(1 - \phi_{12}^2)x_1x_2
\]

\[
> 0.
\] (21)

The inverse of the Jacobian matrix, \( \det(D_v G) \) \( D_v G^{-1} \), is given by

\[
\begin{pmatrix}
L_3 + \phi_{13}z_1 + (1 - \varepsilon)\phi_{23}z_2 + \varepsilon x_2 & -\varepsilon(\phi_{12} - \phi_{23}v_1)x_2 \\
-\varepsilon(\phi_{12} - \phi_{13}v_2)x_1 & L_3 + (1 - \varepsilon)\phi_{13}z_1 + \phi_{23}z_2 + \varepsilon x_1
\end{pmatrix}.
\] (22)

(i) By applying the implicit function theorem to the relative wage equilibrium conditions (17), we get

\[
\frac{dv}{dL_2} = -(D_v G)^{-1}D_{L_2} G = (D_v G)^{-1}v_2^{-\varepsilon} \begin{pmatrix}
\phi_{12} - \phi_{23}v_1 \\
1 - \phi_{23}v_2
\end{pmatrix}
\]

\[
\frac{d(v_2/v_3)}{dL_2} = v_2^{-\varepsilon} [D_v G_{21}^{-1}(\phi_{12} - \phi_{23}v_1) + D_v G_{22}^{-1}(1 - \phi_{23}v_2)]
\]

\[
= \frac{v_2^{-\varepsilon}}{\det(D_v G)} g_1(v_1, v_2),
\]

where

\[
g_1(v_1, v_2) \equiv (1 - \phi_{23}v_2)(1 + \varepsilon x_1 - \varepsilon \phi_{13}z_1) - \varepsilon x_1(\phi_{12} - \phi_{13}v_2)(\phi_{12} - \phi_{23}v_1).
\]
We show that \( g_1(v_1, v_2) > 0 \) in the interior of triangle \( \Delta \). This is to equivalent to show that the curve \( \Gamma_1 \) defined by the expression \( g_1(v_1, v_2) = 0 \) does not intersect triangle \( \Delta \), except at corner \((\phi_{12}/\phi_{23}, \phi_{23}^{-1})\). For this purpose, we evaluate \( g_1 \) along the three sides of triangle \( \Delta \).

First, solving \( l_1(v_1, v_2) = 0 \) for \( v_2 \) and plugging the solution into \( g_1 \) lead to

\[
\frac{g_1(v_1, v_2)}{l_1=0} = \frac{(\phi_{23}v_1 - \phi_{12})(\phi_{13} - \phi_{12}\phi_{23})}{\phi_{23} - \phi_{12}\phi_{13}}.
\]

Because \( v_1 \in [\phi_{12}/\phi_{23}, \phi_{13}^{-1}] \) on the side \((l_1 = 0)\) of triangle \( \Delta \), we get \( g_1(v_1, v_2)|_{l_1=0} \geq 0 \), where the equality holds only if \( v_1 = \phi_{12}/\phi_{23} \).

Second, solving \( l_2(v_1, v_2) = 0 \) for \( v_2 \) and plugging the solution into \( g_1 \) lead to

\[
\frac{g_1(v_1, v_2)}{l_2=0} = \frac{(\phi_{12} - \phi_{23}v_1)(1 - \phi_{23}^2 + \varepsilon x_1 \det \Phi_3)}{\phi_{12} - \phi_{13}\phi_{23}}.
\]

Because \( v_1 \in [\phi_{13}, \phi_{12}/\phi_{23}] \) on the side \((l_2 = 0)\) of triangle \( \Delta \), we get \( g_1(v_1, v_2)|_{l_2=0} \geq 0 \), where the equality holds only if \( v_1 = \phi_{12}/\phi_{23} \).

Third, solving \( l_3(v_1, v_2) = 0 \) for \( v_2 \) and plugging the solution into \( g_1 \) lead to

\[
\frac{g_1(v_1, v_2)}{l_3=0} = \frac{(1 - \phi_{13}v_1)[(\phi_{12} - \phi_{13}\phi_{23})\phi_{23} + \varepsilon \phi_{13}x_1 \det \Phi_3]}{\phi_{13}(1 - \phi_{13}^2)} + \frac{\phi_{13} - \phi_{12}\phi_{23}}{\phi_{13}} > 0
\]

for all \( v_1 \in [\phi_{13}, \phi_{13}^{-1}] \) on the side \((l_3 = 0)\) of triangle \( \Delta \).

Hence, \( g_1 \) is positive along the three sides of triangle \( \Delta \) except at the vertex \((1/\phi_{12}, \phi_{23}/\phi_{12})\).

By a continuity argument, \( g_1 \) is positive inside \( \Delta \).

(ii) Similarly, we get

\[
\frac{d}{dL_1} = -(DvG)^{-1}DL_1G = (DvG)^{-1}v_1^{-\varepsilon} - \frac{1 - \phi_{13}v_1}{\phi_{12} - \phi_{13}v_2}
\]

Then,

\[
\frac{d(v_2/v_3)}{dL_1} = v_1^{-\varepsilon}[DvG^{-1}_{21}(1 - \phi_{13}v_1) + DvG^{-1}_{22}(\phi_{12} - \phi_{13}v_2)]
\]

\[
= \frac{v_1^{-\varepsilon}(\phi_{12} - \phi_{13}v_2)}{\det(DvG)} (L_3 + \phi_{13}z_1 + \phi_{23}z_2)
\]
Appendix B: Proofs of Proposition 3 and Corollary 1

Regarding the proof of Proposition 3, it suffices to show that \( dU_1/dL_1 > 0 \) and \( dU_3/dL_1 > 0 \) as the other results follow from a symmetric argument. By applying the implicit function theorem to the utility equilibrium equations (15), we get \( D_{L_1}v = -(D_vF)^{-1}D_{L_1}F = (D_vF)^{-1}(\frac{z_1}{L_1})' \).

First, we derive the inverse of the Jacobian matrix \( D_vF^{-1} \). Direct computations allow to write \( \det(D_vF)D_vF^{-1} \) as

\[
\begin{pmatrix}
1 + \varepsilon x_2 + x_3 + \varepsilon^2(1 - \phi_{23})x_2x_3 & -\varepsilon x_2[\phi_{12} + \varepsilon(\phi_{12} - \phi_{13}\phi_{23})x_3] & -\varepsilon x_3[\phi_{13} + \varepsilon(\phi_{13} - \phi_{12}\phi_{23})x_2] \\
-\varepsilon x_1[\phi_{12} + \varepsilon(\phi_{12} - \phi_{13}\phi_{23})x_3] & 1 + \varepsilon (x_3 + x_1) + \varepsilon^2(1 - \phi_{13})x_3x_1 & -\varepsilon x_3[\phi_{23} + \varepsilon(\phi_{23} - \phi_{12}\phi_{13})x_1] \\
-\varepsilon x_1[\phi_{13} + \varepsilon(\phi_{13} - \phi_{12}\phi_{23})x_2] & -\varepsilon x_2[\phi_{23} + \varepsilon(\phi_{23} - \phi_{12}\phi_{13})x_1] & 1 + \varepsilon (x_1 + x_2) + \varepsilon^2(1 - \phi_{12})x_1x_2
\end{pmatrix}
\]

Second, we get

\[
\frac{dv_1}{dL_1} = \frac{z_1}{L_1}(D_vF^{-1})_{11} + \phi_{12}D_vF^{-1}_{12} + \phi_{13}D_vF^{-1}_{13}
\]

\[
= \frac{z_1}{L_1 \det(D_vF)}[1 + \varepsilon(1 - \phi_{12})x_2 + \varepsilon(1 - \phi_{13})x_3 + \varepsilon^2 x_2x_3 \det \Phi_3] > 0,
\]

where Lemma 2 has been used. We also have

\[
\frac{dv_3}{dL_1} = \frac{z_1}{L_1}(D_vF^{-1})_{31} + \phi_{12}D_vF^{-1}_{32} + \phi_{13}D_vF^{-1}_{33}
\]

\[
= \frac{z_1}{L_1 \det(D_vF)}[\phi_{13} + \varepsilon(\phi_{13} - \phi_{12}\phi_{23})x_2] > 0
\]

by making use of the triangle inequality \( \phi_{13} > \phi_{12}\phi_{23} \).

So as to prove Corollary 1, the trade share \( \lambda_{ij} \) is rewritten by using Lemma 1 as follows

\[
\lambda_{ij} = n_i \left( \frac{p_{ij}}{P_j} \right)^{1-\sigma} = \frac{L_i\phi_{ij}w_i^{1-\sigma}}{\sum_k L_k\phi_{kj}w_k^{1-\sigma}} = \frac{L_i\phi_{ij}w_i^{1-\sigma}}{w_j^{1-\sigma}}.
\]
The above proof of Proposition 3 ensures that \( dw_j/dL_i > 0 \) since \( dv_j/dL_i > 0 \). We then have

\[
\frac{d\lambda_{kj}}{dL_i} = -L_k \phi_{kj} \left( \frac{\sigma - 1}{w_k w_j} \frac{dw_k}{dL_i} + \frac{\sigma}{w_k^{\sigma - 1} w_j^{\sigma + 1}} \frac{dw_j}{dL_i} \right) < 0, \quad \forall j \text{ and } k \neq i.
\]

Moreover, \( d\lambda_{ij}/dL_i = -\sum_{k \neq i} d\lambda_{kj}/dL_i > 0 \) as trade shares add up to 1.

Appendix C: Proof of Proposition 4

It suffices to show that \( d(w_2/w_3)/d\phi_{12} > 0 \) as the other results follow from a symmetric argument. By applying the implicit function theorem to the relative wage equilibrium conditions \((17)\), we have

\[
\frac{dv}{d\phi_{12}} = -(D_v G)^{-1} D_{\phi_{12}} G = (D_v G)^{-1} \begin{pmatrix} z_2 \\ z_1 \end{pmatrix}.
\]

Notice that \( \det(D_v G) D_v G^{-1} \) has already been derived in expression \((22)\) in Appendix A.

Using \( x_i = z_i/v_i \) and \((13)\), we get

\[
\frac{d(v_2/v_3)}{d\phi_{12}} = D_v G^{-1} z_2 + D_v G^{-1} z_1 = \frac{x_2}{\det(D_v G)} g_2(v_1, v_2),
\]

where

\[
g_2(v_1, v_2) \equiv \{-\epsilon \phi_{13} (1 - \phi_{23}^2) v_1^2 + [\det \Phi_3 + \epsilon (1 + \phi_{12}^2 + \phi_{13}^2 - \phi_{23}^2 - 2\phi_{12}\phi_{13}\phi_{23})] v_1 \\
+ \epsilon \phi_{13} (1 - \phi_{13}^2) v_2^2 - 2\epsilon \phi_{12} (1 - \phi_{13}^2) v_2 - \epsilon (\phi_{13} + \phi_{12}\phi_{13} - 2\phi_{12}\phi_{23}) \} (\det \Phi_3)^{-1}.
\]

We show that \( g_2(v_1, v_2) > 0 \) in the interior of triangle \( \Delta \). This is to equivalent to show that the curve \( \Gamma_2 \) defined by the expression \( g_2(v_1, v_2) = 0 \) does not intersect triangle \( \Delta \). For this purpose, we evaluate \( g_2 \) along the three sides of triangle \( \Delta \).

First, solving \( l_1(v_1, v_2) = 0 \) for \( v_2 \) and plugging the solution into \( g_2 \) lead to

\[
g_2(v_1, v_2) \big|_{l_1=0} = \frac{g_3(v_1)}{(\phi_{12} - \phi_{13}\phi_{23})^2},
\]

25
where

\[ g_3(v_1) \equiv \varepsilon \phi_{13} (\phi_{12}^2 + \phi_{13}^2 - 2\phi_{12}\phi_{13}\phi_{23}) + \varepsilon \phi_{13} (1 - \phi_{23}^2) v_1^2 \]

\[ + \left[ (\phi_{12} - \phi_{13}\phi_{23})^2 - \varepsilon (2\phi_{12}^2 + 2\phi_{13}^2 - \phi_{13}^2\phi_{23}^2 - 2\phi_{12}\phi_{13}\phi_{23}) \right] v_1. \]

Because \( g''_3(v_1) > 0 \), \( g'_3(\phi_{13}) > 0 \) and \( g_3(\phi_{13}) > 0 \), we have \( g_3(v_1) > 0 \) for all \( v_1 \in [\phi_{13}, \phi_{12}/\phi_{23}] \) on the side \((l_1 = 0)\) of triangle \( \Delta \).

Second, solving \( l_2(v_1, v_2) = 0 \) for \( v_2 \) and plugging the solution into \( g_2 \) lead to

\[ g_2(v_1, v_2)|_{l_2=0} = g_4(v_1) = \frac{\varepsilon \phi_{13} v_1^2}{1 - \phi_{13}^2} \varepsilon \phi_{13} (\phi_{12}^2 + \phi_{13}^2 - 2\phi_{12}\phi_{13}\phi_{23}) + \varepsilon \phi_{13} (1 - \phi_{23}^2) v_1^2. \]

Because \( g''_4(v_1) < 0 \), \( g_4(\phi_{13}) > 0 \) and \( g_4(\phi_{13}^{-1}) > 0 \), we have \( g_4(v_1) > 0 \) for all \( v_1 \in [\phi_{13}, \phi_{13}^{-1}] \) on the side \((l_2 = 0)\) of triangle \( \Delta \).

Third, solving \( l_3(v_1, v_2) = 0 \) for \( v_2 \) and plugging the solution into \( g_2 \) lead to

\[ g_2(v_1, v_2)|_{l_3=0} = \frac{g_5(v_1)}{(\phi_{12} - \phi_{13}\phi_{23})^2}, \]

where

\[ g_5(v_1) \equiv \varepsilon (\phi_{13} + \phi_{12}\phi_{13} - 2\phi_{12}\phi_{23}) + \varepsilon \phi_{13} (\phi_{13}^2 - \phi_{23}^2) v_1^2 \]

\[ + \left[ (\phi_{23} - \phi_{12}\phi_{13})^2 - \varepsilon (2\phi_{13}^2 - \phi_{23}^2 + \phi_{12}\phi_{13}^2 - 2\phi_{12}\phi_{13}\phi_{23}) \right] v_1. \]

We get \( g_5(\phi_{12}/\phi_{23}) > 0 \) and \( g_5(\phi_{13}^{-1}) > 0 \). Because \( \text{sgn} g''_5(v_1) = \text{sgn}(\phi_{13} - \phi_{23}) \), two cases may arise.

(a) If \( \phi_{13} \leq \phi_{23} \), then \( g_5 \) is concave, and thus \( g_5(v_1) > 0 \) for all \( v_1 \in [\phi_{12}/\phi_{23}, \phi_{13}^{-1}] \) on the side \((l_3 = 0)\) of triangle \( \Delta \).

(b) If \( \phi_{13} > \phi_{23} \), then \( g_5 \) is convex. Solving \( g''_5(v_1) = 0 \) with respect to \( v_1 \) and plugging the solution into \( g_5 \) leads to

\[ g_6(\varepsilon) = \frac{1}{4\varepsilon \phi_{13} (\phi_{13}^2 - \phi_{23}^2)} A_1 \]

\[ A_1 \equiv - (\phi_{23} - \phi_{12}\phi_{13})^2 (\varepsilon^2 + 1) + 2 (2\phi_{13}^2 - \phi_{23}^2 + \phi_{12}\phi_{13}^2 - 2\phi_{12}\phi_{13}\phi_{23}) \varepsilon. \]
Therefore, if \( A_1 > 0 \), then \( g_6(\varepsilon) > 0 \) holds on side \((l_3 = 0)\). Next, we get
\[
g'(\phi_{12}/\phi_{23}) = \frac{1}{\phi_{23} (\phi_{23} - \phi_{12}\phi_{13})} A_2
\]
\[
A_2 \equiv \phi_{23} (\phi_{23} - \phi_{12}\phi_{13}) - (2\phi_{13}^2 - \phi_{23}^2 - 2\phi_{12}\phi_{13}\phi_{23}) \varepsilon.
\]

Therefore, if \( A_2 > 0 \), then the curve \( \Gamma_2 \) does not intersect \((l_3 = 0)\) for all \( v_1 \in [\phi_{12}/\phi_{23}, \phi_{13}^{-1}] \).

We readily have
\[
A_1 > 0 \iff \varepsilon < \varepsilon_1
\]
\[
A_2 > 0 \iff \varepsilon < \varepsilon_2
\]
\[
0 < \varepsilon_1 < \varepsilon_2 < 1
\]

implying that \( A_1 > 0 \) or \( A_2 > 0 \) holds for all \( v_1 \in [\phi_{12}/\phi_{23}, \phi_{13}^{-1}] \). Hence, the curve \( \Gamma_2 \) does not intersect \((l_3 = 0)\) inside triangle \( \Delta \).

**Appendix D: Proof of Proposition 5**

It suffices to show that \( dU_1/d\phi_{12} > 0 \) and \( dU_3/d\phi_{12} < 0 \) as the other results follow from a symmetric argument. By applying the implicit function theorem to the equilibrium equations (15), we get
\[
D_{\phi_{12}} v = -(D_v F)^{-1}D_{\phi_{12}} F = (D_v F)^{-1} \begin{pmatrix} z_2 & z_1 \\ 0 & 0 \end{pmatrix}'.
\]

We make use of expression (23) of \((D_v F)^{-1}\) derived in the proof of Proposition 3 in Appendix B.

First, note that \((D_v F)^{-1}_{ij} = -\varepsilon x_i [\phi_{ij} + \varepsilon (\phi_{ij} - \phi_{ik}\phi_{jk}) x_k]/\det(D_v F) < 0\), for any distinct \( i, j, k \), given the triangle inequality \( \phi_{ij} > \phi_{ik}\phi_{jk} \). This implies that \( d v_3/d\phi_{12} = (D_v F_{31}^{-1} z_2 + D_v F_{32}^{-1} z_1) < 0 \).

Second, \( d v_1/d\phi_{12} = (D_v F_{11}^{-1} z_2 + D_v F_{12}^{-1} z_1) \). Since \( D_v F_{11}^{-1} = \{1 + \varepsilon x_3 + \varepsilon x_2 [1 + \varepsilon (1 - \phi_{23}^2) x_3]\}/\det(D_v F) > 0\), \( d v_1/d\phi_{12} = (D_v F_{11}^{-1} z_2 + D_v F_{12}^{-1} z_1) \) corresponds to the sum of a positive and a negative term. However, by replacing the utility equilibrium conditions (15), \( d v_1/d\phi_{12} \)
can be expressed in terms of $z$’s only

\[
\frac{dv_1}{d\phi_{12}} = \frac{z_2}{v_2v_3 \det(D_vF)}\left\{(1 - \varepsilon)\phi_{12}\phi_{13}z_1^2 + (1 + \varepsilon)\phi_{23}z_2^2 + (1 + \varepsilon)\phi_{23}z_3^2 + [(1 + \varepsilon)\phi_{13} + (1 - \varepsilon)\phi_{12}\phi_{23}]z_1z_2 + [(1 + \varepsilon)^2 + (1 - \varepsilon)^2]\phi_{23}^2]z_2z_3 + [(1 - \varepsilon^2)\phi_{12} + (1 + \varepsilon^2)\phi_{13}\phi_{23}]z_1z_3\right\}.
\]

By direct inspection of this expression, each term in the sum is positive. Hence $dv_1/d\phi_{12} > 0$.

Appendix E: Proof of Proposition 6

From the utility equilibrium conditions (18), the equilibrium map $F$ can be defined by

\[
F(v) = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} - \begin{pmatrix} z_1 + \phi_{12}z_2 + (N - 2)\phi_{13}z_3 \\ \phi_{12}z_1 + z_2 + (N - 2)\phi_{23}z_3 \\ \phi_{13}z_1 + \phi_{23}z_2 + [1 + (N - 3)\phi]z_3 \end{pmatrix} = 0. \tag{24}
\]

The corresponding Jacobian matrix $D_vF$ is given by

\[
\begin{pmatrix} 1 + \varepsilon x_1 & \varepsilon\phi_{12}x_2 & (N - 2)\varepsilon\phi_{13}x_3 \\ \varepsilon\phi_{12}x_1 & 1 + \varepsilon x_2 & (N - 2)\varepsilon\phi_{23}x_3 \\ \varepsilon\phi_{13}x_1 & \varepsilon\phi_{23}x_2 & 1 + \varepsilon[1 + (N - 3)\phi]x_3 \end{pmatrix}, \tag{25}
\]

where $x_i = z_i/v_i$.

By applying the implicit function theorem to the equilibrium equations (24), we get

\[
(D_vF)^{-1}D_{\phi_{12}}F = -(D_vF)^{-1}\begin{pmatrix} z_2 \\ z_1 \\ 0 \end{pmatrix}'.
\]

First, we show that

\[
\det(D_vF) = 1 + \varepsilon[x_1 + x_2 + x_3 + (N - 3)\phi x_3] + \varepsilon^2 g_7(N) + \varepsilon^3 x_1x_2x_3g_8(N) > 0,
\]

where

\[
g_7(N) = x_1[x_2(1 - \phi_{12}^2) + x_3(1 + (N - 3)\phi - (N - 2)\phi_{13}^2) + x_2x_3[1 + (N - 3)\phi - (N - 2)\phi_{23}^2] \\
g_8(N) = 1 - \phi_{12}^2 + (N - 3)\phi(1 - \phi_{12}^2) - (N - 2)(\phi_{13}^2 + \phi_{23}^2) + 2(N - 2)\phi_{12}\phi_{13}\phi_{23}.
\]
Since \( g_7(3) = x_1[x_2(1 - \phi_{12})^2 + x_3(1 - \phi_{13}^2)] + x_2x_3(1 - \phi_{23}^2) > 0 \) and \( g_7(N + 1) - g_7(N) = x_3[x_1(\phi - \phi_{13}^2) + x_2(\phi - \phi_{23}^2)] > 0 \), we have \( g_7(N) > 0 \).

From the proof of Lemma 2, \( g_8(3) = \det \Phi_3 > 0 \) so that

\[
g_8(N + 1) - g_8(N) = \phi(1 - \phi_{12}^2) - \phi_{13}^2 - \phi_{23}^2 + 2\phi_{12}\phi_{13}\phi_{23}
= (\phi - \phi_{13})(\phi_{13} - \phi_{12}\phi_{23}) + (\phi - \phi_{23})(\phi_{23} - \phi_{12}\phi_{13}) + \phi(1 - \phi_{12})(1 - \phi_{13} - \phi_{23} + \phi_{12})
> 0
\]
as \( 1 - \phi_{13} - \phi_{23} + \phi_{12} > (1 - \phi_{13})(1 - \phi_{23}) > 0 \).

Second, as \( dv_3/d\phi_{12} = [(D_v F^{-1})_{31}z_2 + (D_v F^{-1})_{32}z_1] \), we get

\[
\frac{dv_3}{d\phi_{12}} = -\frac{\varepsilon}{\det(D_v F)}[\phi_{13} + (\phi_{13} - \phi_{12}\phi_{23})\varepsilon x_2]x_1z_2 - \varepsilon[\phi_{23} + \varepsilon(\phi_{23} - \phi_{12}\phi_{13})]x_2z_1 < 0
\]
by making use of the triangle inequalities \( \phi_{13} > \phi_{12}\phi_{23} \) and \( \phi_{23} > \phi_{12}\phi_{13} \).

Third, as \( dv_1/d\phi_{12} = [(D_v F^{-1})_{11}z_2 + (D_v F^{-1})_{12}z_1] \), we get

\[
\frac{dv_1}{d\phi_{12}} = \frac{1}{\det(D_v F)}[1 + \varepsilon x_3(1 + (N - 3)\phi)](x_2 + \varepsilon x_2z_2 - \varepsilon x_2z_1\phi_{12}) + (N - 2)\varepsilon^2 x_2x_3\phi_{23}(z_1\phi_{13} - z_2\phi_{23}).
\]

By using the utility equilibrium conditions (24), and substituting \( x_i \) by \( z_i/v_i \), we have

\[
\frac{v_2v_3}{z_2} \frac{dv_1}{d\phi_{12}} = (1 - \varepsilon)\phi_{12}\phi_{13}z_1^2 + (1 + \varepsilon)\phi_{23}z_2^2 + (N - 2)(1 + \varepsilon)[1 + (N - 3)\phi]\phi_{23}z_3^2
+ \{(1 + \varepsilon^2)[1 + (N - 3)\phi]\phi_{12} + (N - 2)(1 + \varepsilon^2)\phi_{13}\phi_{23}\}z_1z_3
+ [(1 + \varepsilon)\phi_{13} + (1 - \varepsilon)\phi_{12}\phi_{23}]z_1z_2 + [(1 + \varepsilon)^2[1 + (N - 3)\phi] + (N - 2)(1 - \varepsilon^2)\phi_{23}^2]z_2z_3.
\]

By inspection of this expression, we get \( dv_1/d\phi_{12} > 0 \). By a symmetry argument, we also have \( dv_2/d\phi_{12} > 0 \).

Appendix F: Proof of Proposition 7
First, we consider the case of symmetric trade relationships. It suffices to show that $dU_3/d\phi_{12} < 0$ and $dU_1/d\phi_{12} > 0$. From the utility equilibrium conditions (19), the equilibrium map $F$ can be defined by

$$F_i(v) \equiv v_i - z_i - \phi \sum_{j \neq i}^N z_j = 0, \quad i = 1, ..., N.$$  \hfill (26)

The corresponding Jacobian matrix $D_v F$ is given by

$$D_v F = \begin{pmatrix}
1 + \varepsilon x_1 & \varepsilon \phi x_2 & \ldots & \varepsilon \phi x_N \\
\varepsilon \phi x_1 & 1 + \varepsilon x_2 & \ddots & \\
\vdots & \varepsilon \phi x_2 & \ddots & \\
\vdots & \vdots & \ddots & \varepsilon \phi x_N \\
\varepsilon \phi x_1 & \ldots & \ldots & 1 + \varepsilon x_N
\end{pmatrix}, \hfill (27)$$

where $x_i = z_i/v_i$.

(i) By applying the implicit function theorem to the equilibrium equations (26), we get $dv_3/d\phi_{12} = z_2D_v F_{31}^{-1} + z_1D_v F_{32}^{-1}$. Any off-diagonal of $D_v F^{-1}$ can be shown to be negative. This because these elements can be written as

$$D_v F_{i,j \neq i}^{-1} = -\frac{\varepsilon \phi x_j}{\det(D_v F)} \prod_{k \neq i,j} [1 + \varepsilon(1 - \phi)x_k], \hfill (28)$$

where it can be shown that $\det(D_v F) > 0$. Hence $dv_3/d\phi_{12} < 0$.

(ii) Similarly, $dv_1/d\phi_{12} = z_2D_v F_{11}^{-1} + z_1D_v F_{12}^{-1}$ holds. We have

$$D_v F_{11}^{-1} = \frac{1 + \varepsilon x_2}{\det(D_v F)} \prod_{i \neq 1,2} [1 + \varepsilon(1 - \phi)x_i] + \sum_{j \neq 1,2} \frac{\varepsilon \phi x_j}{\det(D_v F)} \prod_{k \neq 1,j} [1 + \varepsilon(1 - \phi)x_k] \hfill (29)$$

$$> \frac{1 + \varepsilon x_2}{\det(D_v F)} \prod_{i \neq 1,2} [1 + \varepsilon(1 - \phi)x_i].$$
From expressions (28) and (29), we have

\[
\frac{d v_1}{d \phi_{12}} > z_2 \frac{1 + \varepsilon x_2}{\det(D_v F)} \prod_{i \neq 1, 2} [1 + \varepsilon (1 - \phi) x_i] - z_1 \varepsilon x_2 \frac{1}{\det(D_v F)} \prod_{k \neq 1, 2} [1 + \varepsilon (1 - \phi) x_k]
\]

\[
= [z_2 (1 + \varepsilon x_2) - z_1 \varepsilon x_2] \prod_{i \neq 1, 2} \frac{1}{\det(D_v F)} [1 + \varepsilon (1 - \phi) x_i]
\]

\[
= \left[ \phi (1 - \varepsilon) z_1 + \phi \sum_{j \neq 1, 2} z_j + (1 + \varepsilon) z_2 \right] \frac{z_2}{v_2 \det(D_v F)} \prod_{i \neq 1, 2} [1 + \varepsilon (1 - \phi) x_i]
\]

\[
> 0,
\]

where the last equality is due to \( x_i = z_i / v_i \) and equation (26).

Second, under sufficiently high differentiation of varieties \((\varepsilon \to 0)\), we have

\[
D_v F^{-1}_{i,j \neq i} = \frac{-\varepsilon x_j + O(\varepsilon^2)}{1 + \varepsilon \sum_k x_k + O(\varepsilon^2)} < 0,
\]

where \( x_i = z_i / v_i \). This implies that \( d v_k / d \phi_{ij} < 0 \).

We also have

\[
D_v F^{-1}_{i,i} = \frac{1 + \varepsilon \sum_{k \neq i} x_k + O(\varepsilon^2)}{1 + \varepsilon \sum_k x_k + O(\varepsilon^2)} \approx 1.
\]

Hence

\[
\frac{d v_i}{d \phi_{ij}} = z_j D_v F^{-1}_{ii} + z_i D_v F^{-1}_{ij}
\]

\[
\approx z_j > 0.
\]

Finally, under sufficiently low differentiation of varieties \((\varepsilon \to 1)\), we get \( \sigma \to \infty \) and \( \phi_{ij} \to 0 \) for all \( i \neq j \). Defining \( \phi_{ij} \equiv \delta \phi_{ij}' \) and letting \( \delta \to 0 \) lead to

\[
D_v F^{-1}_{i,j \neq i} = \frac{-\delta \phi_{ij}' x_i \prod_{k \neq i,j} (1 + x_k) + O(\delta^2)}{\prod_{i=1}^{N} (1 + x_i) + O(\delta^2)} < 0,
\]

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which implies that $dv_k/d\phi_{ij} < 0$. We also have

$$D_v F_{i,i}^{-1} = \prod_{j \neq i}^{N} (1 + x_j) + O(\delta^2) / \prod_{i=1}^{N} (1 + x_i) + O(\delta^2) > 0.$$ 

Hence, $dv_i/d\phi_{ij} = z_j D_v F_{ii}^{-1} + z_i D_v F_{ij}^{-1} > 0$.

References


